REAL NUMBERS

Q. If HCF of 144 and 180 is expressed in the form 13m - 16. Find the value of m.

[Board 2020]

Solution

Thus HCF of 144 and 180 is 36.

Now

$$\begin{aligned} 36 &= 13m - 16 \\ 36 + 16 &= 13m \\ 52 &= 13m \Rightarrow m = 4 \end{aligned}$$

Q. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

[Board 2017]

Solution

$$
\begin{aligned}
3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\
&= 4(909 + 1) \\
&= 4(910) \\
&= 2 \times 2 \times (10 \times 7 \times 13) \\
&= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\
&= \text{ a composite number}
\end{aligned}
$$

Q. Write whether on simplification $\dfrac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ **gives an irrational or a rational number.**

[Board 2018]

Solution

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}=\frac{2\sqrt{3.3.5}+3\sqrt{2.2.5}}{2\sqrt{5}}$$

$$=\frac{2(3\sqrt{5})+3(2\sqrt{5})}{2\sqrt{5}}=\frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}=\frac{12\sqrt{5}}{2\sqrt{5}}=6$$

$\therefore\ \dfrac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ **is a rational number.**

Q. Explain why $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number.

[Board 2015]

Solution

$$17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11 \quad \ldots(i)$$

$$= 2 \times 11 \times (17 \times 5 \times 3 + 1)$$

$$= 2 \times 11 \times (255 + 1) = 2 \times 11 \times 256$$

Number (*i*) is divisible by 2, 11 and 256 and it has more than 2 prime factors.

$\therefore$ $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number.

Q. Find the prime factorisation of the denominator of the rational number expressed as $6.\overline{12}$ in simplest form.

[Board 2014]

Solution

Let $x = 6.\overline{12}$ ...(*i*)

$100x = 612.\overline{12}$ ...(*ii*)

...[Multiplying both sides by 100

Subtracting (*i*) from (*ii*),

$99x = 606 \Rightarrow x = \frac{606}{99} = \frac{202}{33}$

$\therefore$ Denominator = 33

Prime factorisation = 3 × 11

Q. Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

[Board 2012]

Solution

$2053 - 5 = 2048$

$967 - 7 = 960$

Prime factors of 2048 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Prime factors of 960 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$

HCF of 2048 and 960 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Q. If $xy = 180$ and $HCF(x, y) = 3$, then find the $LCM(x, y)$.

[SQP 2021]

Solution

Given. $xy = 180;\qquad HCF(x, y) = 3$

As we know, LCM × HCF = Product of two nos.

$$\therefore\ \mathbf{LCM}(x, y) = \frac{xy}{HCF(x, y)} = \frac{180}{3} = \mathbf{60}$$

Q. 3 bells ring at an interval of 4, 7 and 14 minutes. All three bells tang at 6 a.m., when will the three bells ring together next?

[Board 2013]

Solution

Here $4 = 2^2$

$7 = 7$

$14 = 2 \times 7$

LCM of 4, 7 and 14 is $2^2 \times 7 = 28$ min

$\therefore$ **Three bells will ring together again at 6:28 am.**

Q. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

[Board 2013]

Solution

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $$\sqrt{p} = \frac{m}{n},$$

where m and n are co-primes and $n \neq 0$.
Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$

or, $$pn^2 = m^2 \qquad ...(1)$$

Here p divides pn^2. Thus p divides m^2 and in result p also divides m.
Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2q^2$$

or, $$n^2 = pq^2$$

Solution

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[p is prime and p divides $n^2 \Rightarrow p$ divides n]

Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational. Hence, $\sqrt{p}$ is irrational.

Q. Prove that $\sqrt{3}$ is an irrational number.

[Board 2020]

Solution

Assume that $\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{a}{b}$ where a and b are co-prime integers and $q \neq 0$.

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

Now $$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 3b^2$ we have

$$3b^2 = 9c^2$$

Solution

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b. Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Q. Show that $5\sqrt{6}$ is an irrational number, given $\sqrt{6}$ is an irrational number.

[Board 2015]

Solution

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

Now $$5\sqrt{6} = \frac{a}{b}$$

$$\sqrt{6} = \frac{a}{5b}$$

or, $$\sqrt{6} = \text{rational}$$

But, 4×1 is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number.

Q. Prove that $3 + \sqrt{5}$ is an irrational number.

[Board 2010]

Solution

Assume that $3+\sqrt{5}$ is a rational number, then we have

$$3+\sqrt{5} = \frac{p}{q}, \quad q \neq 0$$

$$\sqrt{5} = \frac{p}{q} - 3$$

$$\sqrt{5} = \frac{p-3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3+\sqrt{5}$ is an irrational number.

Q. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

[Board 2019]

Solution

Assume that $\frac{2+\sqrt{3}}{5}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$\begin{aligned} \frac{2+\sqrt{3}}{5} &= \frac{p}{q} \\ 2+\sqrt{3} &= \frac{5p}{q} \\ \sqrt{3} &= \frac{5p}{q} - 2 \\ \sqrt{3} &= \frac{5p-2q}{q} \end{aligned}$$

Since, p and q are co-prime integers, then $\frac{5p-2q}{q}$ is a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong. Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number.

Q. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

[Board 2020]

Solution

Assume that $2\sqrt{5}-3$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $$2\sqrt{5}-3 = \frac{p}{q}$$

where $q \neq 0$ and p and q are co-prime integers.
Rewriting the above expression as,

$$2\sqrt{5} = \frac{p}{q}+3$$

$$\sqrt{5} = \frac{p+3q}{2q}$$

Here $\frac{p+3q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{5}$ should be a rational number. But $\sqrt{5}$ is irrational. This contradicts the given fact that $\sqrt{5}$ is irrational. Hence $2\sqrt{5}-3$ is an irrational number.

Q. Write the smallest number which is divisible by both 306 and 657.

[Board 2019]

Solution

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Here, the given numbers are 306 and 657.

$$\begin{aligned} 306 &= 6 \times 51 = 3 \times 2 \times 3 \times 17 \\ 657 &= 9 \times 73 = 3 \times 3 \times 73 \\ \text{LCM}(306, 657) &= 2 \times 3 \times 3 \times 17 \times 73 \\ &= 22338 \end{aligned}$$

Hence, the smallest number which is divisible by 306 and 657 is 22338.

Q. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

[Board 2011]

Solution

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$
\begin{aligned}
9 &= 3 \times 3 = 3^2 \\
12 &= 2 \times 2 \times 3 = 2^2 \times 3 \\
15 &= 3 \times 5 \\
\text{LCM}(9,\ 12,\ 15) &= 2^2 \times 3^2 \times 5 \\
&= 180 \text{ minutes}
\end{aligned}
$$

The bells will toll next together after 180 minutes.

Q. Find the HCF and LCM of 510 and 92 and verify that HCF × LCM = Product of two given numbers.

[Board 2011]

Solution

Finding prime factor of given number we have,

$$92 = 2^2 \times 23$$

$$510 = 30 \times 17 = 2 \times 3 \times 5 \times 17$$

$$\text{HCF } (510, 92) = 2$$

$$\text{LCM } (510, 92 = 2^2 \times 23 \times 3 \times 5 \times 14$$

$$= 23460$$

$$\text{HCF } (510, 92) \times \text{LCM } (510, 92)$$

$$= 2 \times 23460 = 46920$$

Product of two numbers $= 510 \times 92 = 46920$

Hence, $\quad \text{HCF} \times \text{LCM} = \text{Product of two numbers}$

Q. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

[Board 2016]

Solution

Here we have to determine the HCF of all length which can measure all dimension.

$$
\begin{aligned}
\text{Length,} \quad l &= 8 \text{ m } 50 \text{ cm} = 850 \text{ cm} \\
&= 50 \times 17 = 2 \times 5^2 \times 17 \\
\text{Breadth,} \quad b &= 6 \text{ m } 25 \text{ cm} = 625 \text{ cm} \\
&= 25 \times 25 = 5^2 \times 5^2 \\
\text{Height,} \quad h &= 4 \text{ m } 75 \text{ cm} = 475 \text{ cm} \\
&= 25 \times 19 = 5^2 \times 19 \\
\text{HCF}(l, b, h) &= \text{HCF } (850, 625, 475) \\
&= \text{HCF } (2 \times 5^2 \times 17, \ \ 5^2, \ \ 5^2 \times 19) \\
&= 5^2 = 25 \text{ cm}
\end{aligned}
$$

Thus 25 cm rod can measure the dimensions of the room exactly. This is longest rod that can measure exactly.

Q. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets.

[Board 2016]

Solution

Required answer is the HCF of 990 and 945.
By using Euclid's Division Lemma, we have

$$990 = 945 \times 1 + 45$$

$$945 = 45 \times 21 + 0$$

Thus HCF of 990 and 945 is 45. The fruit vendor should put 45 fruits in each basket to have minimum number of baskets.

Q. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

[Board 2020]

Solution

Let the number of columns be x which is the largest number, which should divide both 612 and 48. It means x should be HCF of 612 and 48.
According to Euclid's algorithm any number a can be written in the form,

$$a = bq + r \text{ where } 0 \leq r < b$$

By using Euclid's Division Lemma, we have

$$612 = 48 \times 12 + 36$$

$$48 = 36 \times 1 + 12$$

$$36 = 12 \times 3 + 0$$

Thus HCF of 612 and 96 is 12 i.e. 12 columns are required.

Q. There are 44 boys and 32 girls in a class. These students arranged in rows for a prayer in such a way that each row consists of only either boys or girls, and every row contains an equal number of students. Find the minimum number of rows in which all students can be arranged.

Solution

$44 = 2^2 \times 11$

$32 = 2^5$

HCF = $2^2 = 4$

Therefore, minimum number of rows in which all students can be arranged = 44/4 + 32/4

= 11 + 8 = 19 rows

Q. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

[Board 2011]

Solution

The required answer will be HCF of 144 and 90.

$$144 = 2^4 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

Thus each stack would have 18 cartons.

Q. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

[Board 2019]

Solution

Assume that $2+5\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$\begin{aligned} 2+5\sqrt{3} &= \frac{p}{q}, \quad q \neq 0 \\ 5\sqrt{3} &= \frac{p}{q} - 2 \\ 5\sqrt{3} &= \frac{p-2q}{q} \\ \sqrt{3} &= \frac{p-2q}{5q} \end{aligned}$$

Here $\sqrt{3}$ is irrational and $\frac{p-2q}{5q}$ is rational because p and q are co-prime integers. But rational number cannot be equal to an irrational number. Hence $2+5\sqrt{3}$ is an irrational number.

Q. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

[Board 2015]

Solution

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\text{HCF} = 24$$

$$\text{LCM} = 540$$

$$\frac{LCM}{HCF} = \frac{540}{24} = 22.5 \text{ not an integer}$$

Q. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF × LCM of these numbers equal to the product of the given three numbers?

[Board 2009]

Solution

Finding prime factor of given number we have,

$$378 = 2 \times 3^3 \times 7$$

$$180 = 2^2 \times 3^2 \times 5$$

$$420 = 2^2 \times 3 \times 7 \times 5$$

$$\text{HCF}(378,\ 180,\ 420) = 2 \times 3 = 6$$

$$\text{LCM}(378,\ 180,\ 420) = 2^2 \times 3^3 \times 5 \times 7$$

$$= 2^2 \times 3^3 \times 5 \times 7 = 3780$$

$$\text{HCF} \times \text{LCM} = 6 \times 3780 = 22680$$

Product of given numbers

$$= 378 \times 180 \times 420$$

$$= 28576800$$

Hence, HCF $\times$ LCM $\neq$ Product of three numbers.

Q. The sum of HCF and LCM of two numbers is 7380. If the LCM of these numbers is 7340 more than their HCF. Find the product of the two numbers.

Solution

We know that,
L.C.M. ❌ H.C.F. = product of those two no.s.
Let L.C.M. be x and H.C.F by y
$x + y = 7380$ ------------ (1)
$x = y + 7340$ ------------ (2)
By Substitution
$(y + 7340) + y = 7380$
$y = 20 \Rightarrow$ HCF = 20 & LCM = 7360
We all know that
Product of Numbers = Product of their H.C.F
Product of Numbers = $20 \times 7360 = 147200$

Q. Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Solution

Ans: Given, two brands of chocolates are available in pack of 24 and 15 respectively. We need to buy an equal number of chocolates of both kinds.

For this we find the LCM of 24 and 15

Now,

$24 = 2^3 \times 3$

$15 = 3 \times 5$

$LCM(24, 15) = 2^3 \times 3 \times 5 = 120$

Now,

$$24 = \frac{120}{24} = 5 \qquad 15 = \frac{120}{15} = 8$$

Therefore, the least number of boxes are 5 of first kind and 8 of second kind should be purchased.

Q. Find the greatest 6-digit number which is exactly divisible by 15, 24 and 36.

Solution

The required greatest 6-digit number is a multiple of LCM (24, 15, 36).

Now, $24 = 2^3 \times 3$;

$15 = 3 \times 5$;

$36 = 22 \times 32$

LCM (24, 15, 36) = $23 \times 32 \times 5 = 360$

Greatest 6-digit number is 999999.

Now, We will divide this number by LCM, we will get, 999999/360 = 2777.775

Now we will multiply 2777 by LCM i.e. 360, we will get, 999720.

So, the required number is 999720.

Q. Explain why 13233343563715 is a composite number?

[Board 2016]

Solution

The number 13233343563715 ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

Q. Find HCF of the numbers given below: k, 2k, 3k, 4k and 5k, where k is a positive integer.

[Board 2015]

Solution

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of $k, 2k, 3k, 4k$ and $5k$, is k.

Q. Three sets of Mathematics, Science and Biology books have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. The number of Mathematics books is 240, the number of Science books is 960 and the number of Biology books is 1024. The number of stack of Mathematics, Science and Biology books, assuming that the books are of the same thickness are respectively.

Solution

The prime factorisation of 240, 960 and 1024 is given below:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3 \times 5$$

$$960 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$$
$$= 2^6 \times 3 \times 5$$

and $$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^{10}$$

HCF of 240, 960 and 1024 $= 2^4 = 16$

Hence, there must be 16 books in each stack.

Now, number of stacks of Mathematics books

$$= \frac{240}{16} = 15$$

Number of stacks of Science books

$$= \frac{960}{16} = 60$$

and, number of stacks of Biology books

$$= \frac{1024}{16} = 64$$

Q. A circular field has a circumference of 360 km. Two cyclists Sumeet and John start together and can cycle at speeds of 12 km/h and 15 km/h respectively, round the circular field. They will meet again at the starting point after how long?

Solution

Given, $\quad$ Total distance $= 360$ km

and, $\quad$ Speed of Sumeet $= 12$ km/h

Number of hours taken by Sumeet to complete 1 round.

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{12}$$

$$= 30 \text{ h}$$

and, $\quad$ Speed of John $= 15$ km/h

Number of hours taken by John to complete 1 round

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{15}$$

$$= 24 \text{ h}$$

Solution

Thus, Sumeet and John complete 1 round in 30 h and 24 h, respectively.
Now, to find required hours, we find the LCM of 30 and 24.

$$\begin{aligned} 30 &= 2 \times 3 \times 5 \\ 24 &= 2 \times 2 \times 2 \times 3 \end{aligned}$$

Then, $$\begin{aligned} \text{LCM } (30, 24) &= 2 \times 2 \times 2 \times 3 \times 5 \\ &= 120 \end{aligned}$$

Hence, Sumeet and John will meet each other again after 120 h.

Q. The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is

Solution

Since required number is largest, the problem is related to HCF. Since, 5 and 8 are the remainders from the numbers, we have the numbers $65 = (70 - 5)$, $117 = (125 - 8)$, which is divisible by the required number.

Now, required number $= \text{HCF}(65, 117)$

By Euclid's division algorithm,

$$117 = 65 \times 1 + 52$$

$$65 = 52 \times 1 + 13$$

$$52 = 13 \times 4 + 0$$

$$\text{HCF } (65, 117) = 13$$

Hence, 13 is the largest number which divides 70 and 125 leaving remainders 5 and 8.

Q. There are 104 students in class 10 and 96 students in class 9 in a school. In a house examination the students are to be evenly seated in parallel rows such that no 2 adjacent rows are of the same class.

a. Find the maximum number of parallel rows of each class for the seating arrangement.

b. Also find the number of students in class 9 and also class 10 in a row.

[Board 2013]

Solution

We find H.C.F. of 104 and 96

$104 = 2^3 \times 13$

2	104
2	52
2	26
	13

2	96
2	48
2	24
2	12
2	6
	3

$96 = 2^5 \times 3$

$\therefore$ **HCF** $= 2^3 =$ **8**

There are 8 students in a row.

(*a*) Number of rows of students of class X

$$= \frac{104}{8} = 13$$

Number of rows of students of class IX

$$= \frac{96}{8} = 12$$

$\therefore$ Total number of rows = 13 + 12 = **25**

(*b*) No. of students of class IX in a row = **8**

No. of students of class X in a row = **8**

POLYNOMIALS

Q. Find the quadratic polynomial, the sum of whose zeroes is −5 and their product is 6.

[Board 2020]

Solution

Let α and β be the zeroes of the quadratic polynomial, then we have

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

Now $$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

Q. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the what is the value of k ?

[Board 2020]

Solution

We have $p(x) = x^2 + 3x + k$

If 2 is a zero of $p(x)$, then we have

$$p(2) = 0$$

$$(2)^2 + 3(2) + k = 0$$

$$4 + 6 + k = 0$$

$$10 + k = 0 \Rightarrow k = -10$$

Q. What are the zeroes of the polynomial $x^2 - 3x - m(m + 3)$?

[Board 2020]

Solution

We have $\quad p(x) = x^2 - 3x - m(m+3)$

Substituting $x = -m$ in $p(x)$ we have

$$p(-m) = (-m)^2 - 3(-m) - m(m+3)$$
$$= m^2 + 3m - m^2 - 3m = 0$$

Thus $x = -m$ is a zero of given polynomial.

Now substituting $x = m+3$ in given polynomial we have

$$p(x) = (m+3)^2 - 3(m+3) - m(m+3)$$
$$= (m+3)[m+3-3-m]$$
$$= (m+3)[0] = 0$$

Thus $x = m+3$ is also a zero of given polynomial.

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.

Q. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

[Board 2020]

Solution

Sum of zeroes, $\alpha+\beta = 6$

Product of zeroes $\alpha\beta = 9$

Now $p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9 = (x-3)(x-3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3.

Q. If p and q are the zeroes of polynomial f(x) = $2x^2 - 7x + 3$, find the value of $p^2 + Q$.

Solution

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

Since, $(p+q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p+q)^2 - 2pq$

$$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$$

Hence $p^2 + q^2 = \frac{37}{4}$.

Q. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

[Board 2012]

Solution

We have $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $\alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

Sum of zeroes $0 = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

Q. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k + 1)$ has sum of its zeros equal to half of their product.

[Board 2019]

Solution

Let α and β be the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k+1) = 0$$

Now sum of roots,. $\alpha + \beta = -\frac{-(k+6)}{1}$

$$= k+6$$

Product of roots, $\alpha\beta = \frac{2(2k+1)}{1}$

$$= 2(2k+1)$$

According to given condition,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$k+6 = \frac{1}{2}[2(2k+1)]$$

$$k+6 = 2k+1 \Rightarrow k = 5$$

Hence, the value of k is 5.

Q. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

[Board 2011]

Solution

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

Q. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

[Board 2020]

Solution

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

Q. Find the zeroes of the quadratic polynomial $5x^2 + 8x - 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

[Board 2013]

Solution

We have $p(x) = 5x^2 + 8x - 4 = 0$

$$= 5x^2 + 10x - 2x - 4 = 0$$

$$= 5x(x+2) - 2(x+2) = 0$$

$$= (x+2)(5x-2)$$

Substituting $p(x) = 0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes} = -2 + \frac{2}{5} = \frac{-8}{5}$$

$$\text{Product of zeroes} = (-2) \times \left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

Sum of zeroes $-\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-8}{5}$

Product of zeroes $\frac{c}{a} = \frac{\text{Constan term}}{\text{Coefficient of } x^2} = -\frac{4}{5}$

Hence Verified.

Q. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = 21/4$, then find the value of k.

[Board 2012]

Solution

We have $p(x) = 2x^2 + 5x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{k}{2}$

According to the question,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - \alpha\beta = \frac{21}{4}$$

$$(\alpha + b)^2 - \alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4}$$

$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

Q. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

[Board 2019]

Solution

We have $$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

$$21y^2 - 11y - 2 = 0 \quad \text{...(1)}$$

$$21y^2 - 14y + 3y - 2 = 0$$

$$7y(3y - 2) + (3y - 2) = 0$$

$$(3y - 2)(7y + 1) = 0$$

$$y = \frac{2}{3}, \frac{-1}{7}$$

Hence, zeros of given polynomial are,

$$y = \frac{2}{3} \text{ and } y = \frac{-1}{7}$$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 21$, $b = -11$ and $c = -2$

Solution

Now, sum of roots, $\alpha+\beta=\frac{2}{3}+\left(-\frac{1}{7}\right)$

$$=\frac{2}{3}-\frac{1}{7}=\frac{11}{21}$$

Thus $\alpha+\beta=-\frac{b}{a}$ Hence verified

and product of roots, $\alpha\beta=\frac{2}{3}\times\left(-\frac{1}{7}\right)=\frac{-2}{21}$

Thus $\alpha\beta=\frac{c}{a}$ Hence verified

Q. If α and β are zeroes of the polynomial $p(x) = 6x^2 - 5x + k$ such that , $\alpha - \beta = 1/6$, find the value of k.

[Board 2007]

Solution

We have $p(x) = 6x^2 - 5x + k$

Since α and β are zeroes of

$$p(x) = 6x^2 - 5x + k,$$

Sum of zeroes, $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha - \beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$.

Q. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x2 - 5x + 7$ find a polynomial whose zeroes are
$2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Solution

When roots are given, equation in quadratic form

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Here roots are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$

$\therefore x^2 - (2\alpha + 3\beta + 3\alpha + 2\beta)x + (2\alpha + 3\beta)(3\alpha + 2\beta) = 0$

$x^2 - 5(\alpha + \beta)x + 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2 = 0$

$x^2 - 5(\alpha + \beta)x + 6 \times (\alpha^2 + \beta^2) + 13\alpha\beta$

$f(x) = 2x^2 - 5x + 7$

$\alpha + \beta = \frac{5}{2}, \alpha\beta = \frac{7}{2}$

$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \frac{25}{4} - 7 = \frac{-3}{4}$

$x^2 - 5\left(\frac{5}{2}\right)x + 6\left(-\frac{3}{4}\right) + 13 \times \frac{7}{2} = 0$

$x^2 - \frac{25}{2}x - \frac{9}{2} + \frac{91}{2} = 0$

Solution

$$x^2 - \frac{25}{2}x - \frac{9}{2} + \frac{91}{2} = 0$$

$$x^2 - \frac{25}{2}x + 41 = 0$$

$$\therefore\ 2x^2 - 25x + 82 = 0$$

Q. If α and β are the roots of the equation ax2 + bx + c = 0 and if px2 + qx + r = 0 has roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, then r equals:

[Board 2020]

Solution

As α, β are the roots of the equation $ax^2 + bx + c = 0$,

so $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

The equation whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$

can be written as:

$$x^2 - \left(\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\right)x + \left(\frac{1-\alpha}{\alpha} \cdot \frac{1-\beta}{\beta}\right) = 0$$

Now, $\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta} = \frac{\beta - \alpha\beta + \alpha - \alpha\beta}{\alpha\beta}$

$$= \frac{\alpha + \beta - 2\alpha\beta}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta} - 2$$

$$= \frac{-b/a}{c/a} - 2$$

$$= -\frac{b}{c} - 2$$

$$= -\frac{b-2c}{c}$$

and $\left(\frac{1-\alpha}{\alpha} \cdot \frac{1-\beta}{\beta}\right) = \frac{1 - \alpha - \beta + \alpha\beta}{\alpha\beta} = \frac{1 - (\alpha + \beta) + \alpha\beta}{\alpha\beta}$

$$= \frac{1}{\alpha\beta} - \frac{\alpha + \beta}{\alpha\beta} + 1 = \frac{a}{c} + \frac{b}{c} + 1 = \frac{a+b+c}{c}$$

Solution

$\therefore$ The required equation is

$$x^2 - \left[-\frac{(b+2c)}{c}\right]x + \left[\frac{a+b+c}{c}\right] = 0$$

$$\Rightarrow cx^2 + (b+2c)x + (a+b+c) = 0 \ldots (i)$$

Compairing eqn. (i) with the given equation $px^2 + qx + r = 0$ we get $r = a + b + c$

Q. If α and β are the zeroes of the polynomial $x^2 = 4x + 3$, find the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

[Board 2020]

Solution

We have $p(x) = x^2 + 4x + 3$

Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$,

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1 + \beta_1 = 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

Solution

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

Hence, required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)x + \alpha_1\beta_1$$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}\left(3x^2 - 16x + 16\right)$$

Q. If α and β are the zeroes the polynomial 2x2 - 4x + 5, find the values of

(i) $\alpha^2 + \beta^2$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iii) $(\alpha - \beta)^2$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(v) $\alpha^3 + \beta^3$

Solution

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5,$ then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 2^2 - 2 \times \frac{5}{2}$$

$$= 4 - 5 = -1$$

Solution

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$$

Solution

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

$$\text{and}\,\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= 2^2 - \frac{4 \times 5}{2}$$

$$= 4 - 10 = -6$$

Solution

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{-1}{\left(\frac{5}{2}\right)^2} = \frac{-4}{25}$$

Solution

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= 2^3 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Q. Find the value of k for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent.

[Board 2020]

Solution

We have $x + 2y - 3 = 0$

and $5x + ky + 7 = 0$

If system is inconsistent, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5} = \frac{2}{k} \Rightarrow k = 10$$

Q. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel?

$3x - y - 5 = 0$

$6x - 2y - p = 0$

[Board 2017]

Solution

We have, $3x - y - 5 = 0$

and $6x - 2y - p = 0$

Here, $a_1 = 3,\ b_1 = -1,\ c_1 = -5$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -p$

Since given lines are parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{-1}{-2} \neq \frac{-5}{-p}$$

$$p \neq 5 \times 2 \Rightarrow p \neq 10$$

Q. The 2 digit number which becomes $\frac{5}{6}$ th of itself when its digits are reversed. If the difference in the digits of the number being 1, what is the two digits number?

[Board 2011]

Solution

If the two digits are x and y, then the number is $10x+y$.

Now $$\frac{5}{6}(10x+y)=10y+x$$

Solving, we get $44x+55y$

$$\frac{x}{y}=\frac{5}{4}$$

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$.
Therefore, number is 54.

Q. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. What is the number?

[Board 2016]

Solution

Let x be units digit and y be tens digit, then number will be $10y+x$

Then, $x = 2y$ ...(1)

If 36 be added to the number, the digits are reversed, i.e number will be $10x+y$.

$$10y+x+36 = 10x+y$$

$$9x-9y = 36$$

$$x-y = 4 \quad ...(2)$$

Solving (1) and (2) we have $x=8$ and $y=4$.

Thus number is 48.

Q. If $3x + 4y : x + 2y = 9 : 4$, then find the value of $3x + 5y : 3x - y$.

[Board 2012]

Solution

We have $$\frac{3x+4y}{x+2y}=\frac{9}{4}$$

Hence, $$12x+16y=9x+18y$$

or $$3x=2y$$

$$x=\frac{2}{3}y$$

Substituting $x=\frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y}=\frac{7y}{y}=\frac{7}{1}=7:1$$

Q. If a pair of linear equations is consistent, then the lines will be intersecting or coincident. Justify.

[Board 2012]

Solution

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ [coincident or dependent]

Q. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

[Board 2011]

Solution

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

Q. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

[Board 2007]

Solution

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

Q. Solve graphically : $2x - 3y + 13 = 0$; $3x - 2y + 12 = 0$

[Board 2020]

Solution

We have $2x - 3y + 13 = 0$

and $3x - 2y + 12 = 0$

Now $2x - 3y = -13$

$$y = \frac{2x + 13}{3}$$

x	0	-6.5	1
y	4.3	0	5

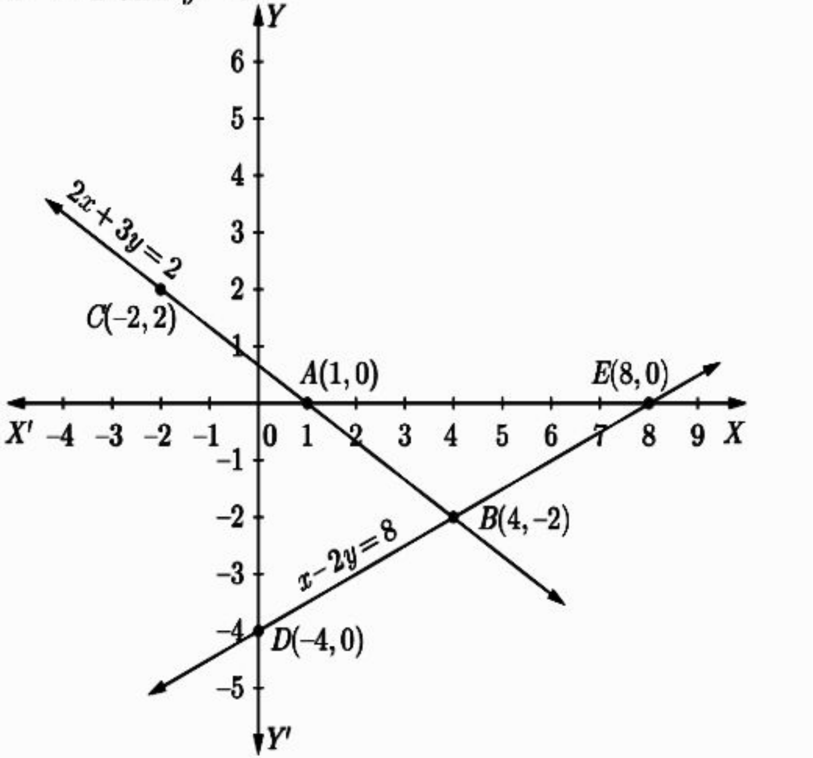
and $3x - 2y = -12$

$$y = \frac{3x + 12}{2}$$

x	0	-4	-2
y	6	0	3

Solution

These lines intersect each other at point $(-2, 3)$ Hence, $x = -2$ and $y = 3$.



Q. Given the linear equation $2x + 3y - 8 = 0$ write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(a) intersecting lines

(b) parallel lines

(c) coincident lines.

[Board 2014]

Solution

Given, linear equation is $2x + 3y - 8 = 0$ ...(1)

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x + 2y - 9 = 0 \qquad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

$$6x + 9y + 7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x + 6y - 16 = 0$$

Q. For what value of k, which the following pair of linear equations have infinitely many solutions:

$2x + 3y = 7$ and $(k + 1)x + (2k - 1)y = 4k + 1$

[Board 2014]

Solution

We have $2x + 3y = 7$

and $(k+1)x + (2k-1)y = 4k+1$

Here $\dfrac{a_1}{a_2} = \dfrac{2}{k+1}, \dfrac{b_1}{b_2} = \dfrac{3}{(2k-1)}$

and $\dfrac{c_1}{c_2} = \dfrac{-7}{-(4k+1)} = \dfrac{7}{(4k+1)}$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\dfrac{a_1}{a_2} = \dfrac{c_1}{c_2}$ we have

$$\frac{2}{k+1} = \frac{7}{4k+1}$$

$$2(4k+1) = 7(k+1)$$

$$8k+2 = 7k+7$$

$$k = 5$$

Hence, the value of k is 5, for which the given equation have infinitely many solutions.

Q. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

[Board 2020]

Solution

We have $2y - x = 8$

L_1 : $x = 2y - 8$

y	0	4	5
$x = 2y - 8$	-8	0	2

$5y - x = 14$

L_2 : $x = 5y - 14$

y	3	4	2
$x = 5y - 14$	1	6	-4

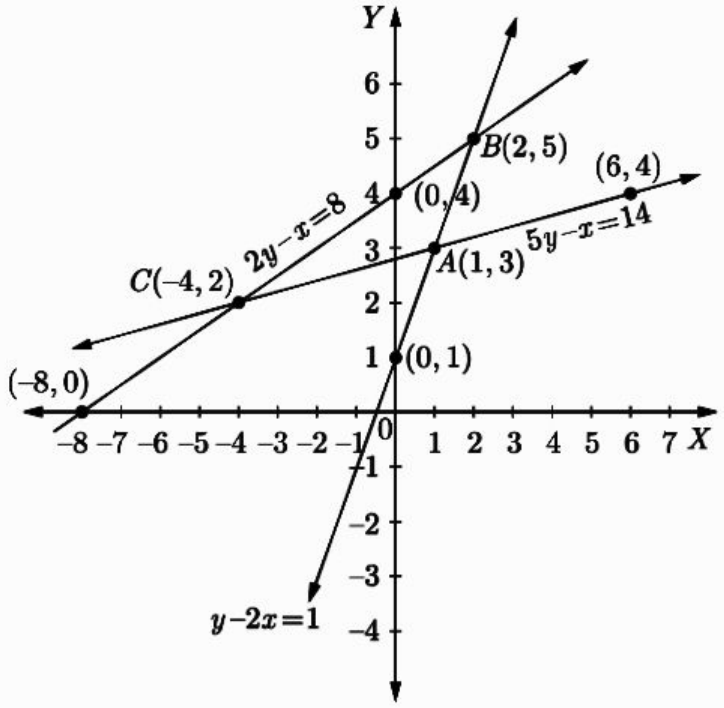
and $y - 2x = 1$

L_3 : $y = 1 + 2x$

x	0	1	2
$y = 1 + 2x$	1	3	5

Solution

Plotting the above points and drawing lines joining them, we get the graphical representation:



Hence, the coordinates of the vertices of a triangle ABC are $A(1,3)$, $B(2,5)$ and $C(-4,2)$.

Q. Aftab tells his daughter, ‘7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.’ Represent this situation algebraically and graphically.

[Board 2020]

Solution

Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Sol :

Let the present age of Aftab be x years and the age of daughter be y years.

$$\text{7 years ago father's(Aftab) age} = (x-7)\text{years}$$

$$\text{7 years ago daughter's age} = (y-7) \text{ years}$$

According to the question,

$$(x-7) = 7(y-7)$$

or,
$$(x-7y) = -42 \quad (1)$$

$$\text{After 3 years father's(Aftab) age} = (x+3) \text{ years}$$

$$\text{After 3 years daughter's age} = (y+3) \text{ years}$$

Solution

According to the condition,

$$x+3 = 3(y+3)$$

or, $$x-3y = 6 \quad (2)$$

From equation(1) $$x-7y = -42$$

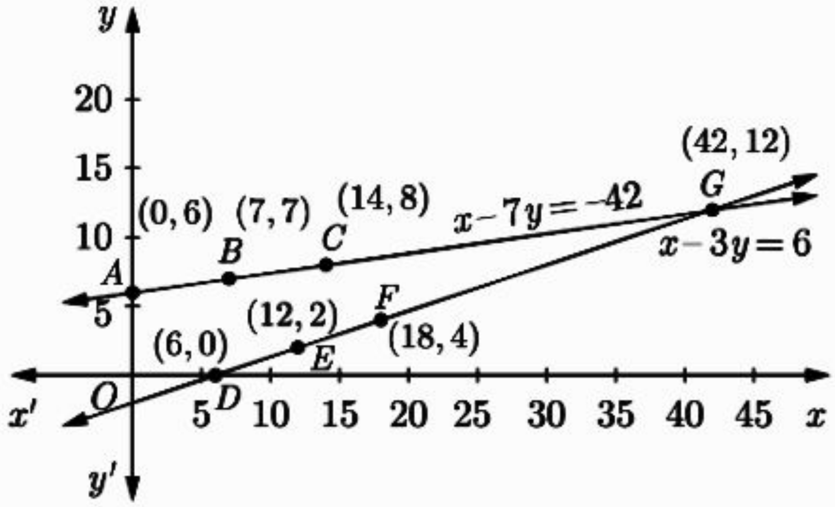
x	0	7	14
$y=\dfrac{x+42}{7}$	6	7	8

From equation (2) $x-3y = 6$

x	6	12	18
$y=\dfrac{x-6}{3}$	0	2	4

Plotting the above points and drawing lines joining them, we get the following graph.

Solution



Two lines obtained intersect each other at $(42, 12)$

Hence, father's age $= 42$ years

and daughter's age $= 12$ years

Q. Solve for x and y:

$$2x - y + 3 = 0 \text{ and } 3x - 5y + 1 = 0$$

[Board 2015]

Solution

We have $2x - y + 3 = 0$ ...(1)

$3x - 5y + 1 = 0$ ...(2)

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$

$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1) we get

$$2x - y + 3 = 0$$

$$2(-2) - y + 3 = 0$$

$$-4 - y + 3 = 0$$

$$-y - 1 = 0$$

$$y = -1$$

Hence, $x = -2$ and $y = -1$.

Q. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr faster it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Solution

Let the actual speed of the train be *s* and actual time taken *t*.

Distance = Speed × Time

$= st$ km

According to the given condition, we have

$$st = (s + 10)(t - 2)$$

$$st = st - 2s + 10t - 20$$

$$2s - 10t + 20 = 0$$

$$s - 5t = -10 \qquad (1)$$

and $$st = (s - 10)(t + 3)$$

$$st = st + 3s - 10t - 30$$

$$3s - 10t = 30 \qquad (2)$$

Solution

Multiplying equation (1) by 3 and subtracting equation

(2) from equation (1),

$$3 \times (s - 5t) - (3s - 10t) = -3 \times 10 - 30$$

$$-5t = -60 \Rightarrow t = 12$$

Substituting value of t equation (1),

$$s - 5 \times 12 = -10$$

$$s = -10 + 60 = 50$$

Hence, the distance covered by the train

$$= 50 \times 12 = 600 \text{ km.}$$

QUADRATIC EQUATIONS

Q.Find the roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant.

(CBSE 2011)

Solution

Solution:

$x^2 - 3x - m(m + 3) = 0$

$D = b^2 - 4ac$

$D = (-3)^2 - 4(1)[-m(m + 3)]$

$= 9 + 4m(m + 3)$

$= 4m^2 + 12m + 9 = (2m + 3)^2$

$$x = \frac{-(-3) \pm \sqrt{(2m+3)^2}}{2(1)} = \frac{3 \pm (2m+3)}{2}$$

$$= \frac{3 + 2m + 3}{2} \text{ or } \frac{3 - 2m - 3}{2}$$

$$x = \frac{2(m+3)}{2} = \frac{-2m}{2}$$

$\therefore x = m + 3$ or $-m$

Q.If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then find the value of ab.

(2012D)

Solution

Solution:

$ay^2 + ay + 3 = 0$

$a(1)^2 + a(1) + 3 = 0$

$2a = -3$

$a = \frac{-3}{2}$

$y^2 + y + b = 0$

$1^2 + 1 + b = 0$

$b = -2$

$\therefore ab = \left(\frac{-3}{2}\right)(-2) = 3$

Q.If $x = -1/2$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

(2015D)

Solution

Solution:

The given quadratic equation can be written as, $3x^2 + 2kx - 3 = 0$

$$3\left(\frac{-1}{2}\right)^2 + 2k\left(\frac{-1}{2}\right) - 3 = 0$$

$$\Rightarrow \frac{3}{4} - k - 3 = 0 \qquad \Rightarrow \qquad -k + \frac{3}{4} - \frac{3}{1} = 0$$

$$\Rightarrow -k + \frac{3-12}{4} = 0 \qquad \Rightarrow \qquad -k - \frac{9}{4} = 0$$

$$\Rightarrow -k = \frac{9}{4} \qquad \therefore \qquad k = \frac{-9}{4}$$

Q.Find the value of p so that the quadratic equation $px(x - 3) + 9 = 0$ has two equal roots. (2011D, 2014OD)

Solution

Solution:

We have, $px(x-3)+9=0$

$px^2-3px+9=0$ Here $a=p$, $b=-3p$,

$D=0$

$b^2-4ac=0 \Rightarrow (-3p)^2-4(p)(9)=0$

$\Rightarrow 9p^2-36p=0$

$\Rightarrow 9p(p-4)=0$

$\Rightarrow 9p=0$ or $p-4=0$

$p=0$ (rejected) or $p=4$

$\therefore p=4$ ....($\because$ Coeff. of x^2 cannot be zero

Q.$36x^2 - 12ax + (a^2 - b^2) = 0$

(2011OD)

Solution

Solution:

We have, $36x^2 - 12ax + (a^2 - b^2) = 0$

$\Rightarrow (36x^2 - 12ax + a^2) - b^2 = 0$

$\Rightarrow [(6x)^2 - 2(6x)(a) + (a)^2] - b^2 = 0$

$\Rightarrow (6x - a)^2 - (b)^2 = 0$ ...[$\because x^2 - 2xy + y^2 = (x - y)^2$

$\Rightarrow (6x - a + b)(6x - a - b) = 0$ „[$\because x^2 - y^2 = (x + y)(x - y)$

$\Rightarrow 6x - a + b = 0$ or $6x - a - b = 0$

$\Rightarrow 6x = a - b$ or $6x = a + b$

$\Rightarrow x = \frac{a-b}{6}$ or $\frac{a+b}{6}$

Q.Solve the following quadratic equation for x:

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$ (2015D)

Solution

Solution:

The given quadratic equation can be written as,

$4x^2 - 4a^2x + (a4^4 - b^4) = 0$

$(4x^2 - 4a^2x + a^4) - b^4 = 0$

or $(2x - a^2)^2 - (b^2)^2 = 0$

$\Rightarrow (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$\Rightarrow (2x - a^2 + b^2) = 0$ or $(2x - a^2 - b^2) = 0$

$\therefore x = \frac{a^2 - b^2}{2}$ or $x = \frac{a^2 + b^2}{2}$

Q.Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

(2017OD)

Solution

Solution:

Given equation is $px^2 - 14x + 8 = 0$.

Here a = p b = -14 c = 8

Let roots be a and 6α.

Sum of roots,

$$\alpha + 6\alpha = \frac{-b}{a}$$

$$7\alpha = \frac{-(-14)}{p}$$

$$\alpha = \frac{2}{p} \quad ...(i)$$

Product of roots,

$$\alpha(6\alpha) = \frac{c}{a}$$

$$6\alpha^2 = \frac{8}{p}$$

$$6\left(\frac{2}{p}\right)^2 = \frac{8}{p} \quad ...[\text{From } (i)$$

$$\frac{24}{p^2} = \frac{8}{p}$$

$$\Rightarrow 8p^2 = 24p$$

$$\Rightarrow 8p^2 - 24p = 0$$

$$\Rightarrow 8p(p - 3) = 0$$

$$\Rightarrow 8p = 0 \text{ or } p - 3 = 0$$

$$\Rightarrow p = 0 \text{ or } p = 3$$

But $p \neq 0$

...[∵ coefficient of $x^2 \neq 0$

$\therefore$ $p = 3$

Q.Solve the equation for x (2014)

$$\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, -\frac{3}{2}$$

Solution

Solution:

$$\frac{4}{x} - \frac{3}{1} = \frac{5}{2x+3}$$

$$\Rightarrow \frac{4-3x}{x} = \frac{5}{(2x+3)}$$

$\Rightarrow 5x = (2x + 3)(4 - 3x)$

$\Rightarrow 5x = 8x - 6x^2 + 12 - 9x$

$\Rightarrow 5x - 8x + 6x^2 - 12 + 9x = 0$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$ ...[Dividing by 6

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - 1(x + 2) = 0$

$\Rightarrow x - 1 = 0$ or $x + 2 = 0$

$\therefore x = 1$ or $x = -2$

Q.Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers. (2016OD)

Solution

Solution:

Let three consecutive natural numbers are x, x + 1, x + 2.

According to the question,

$(x + 1)^2 - [(x + 2)^2 - x^2] = 60$

$\Rightarrow x^2 + 2x + 1 - (x^2 + 4x + 4 - x^2) = 60$

$\Rightarrow x^2 + 2x + 1 - 4x - 4 - 60 = 0$

$\Rightarrow x^2 - 2x - 63 = 0$

$\Rightarrow x^2 - 9x + 7x - 63 = 0$

$\Rightarrow x(x - 9) + 7(x - 9) = 0$

$\Rightarrow (x - 9)(x + 7) = 0$

$\Rightarrow x - 9 = 0$ or $x + 7 = 0$

$\Rightarrow x = 9$ or $x = -7$

Natural nos. can not be -ve, $\therefore x = 9$

$\therefore$ Numbers are 9, 10, 11.

Q.Sum of the areas of two squares is 400 cm2. If the difference of their perimeters is 16 cm, find the sides of the two squares. (2013D)

Solution

Solution:

Let the side of Large square = x cm

Let the side of small square = y cm

According to the Question,

$x^2 + y^2 = 400$... (i) ...[∵ area of square = $(side)^2$

$4x - 4y = 16$ ...[∵ Perimeter of square = 4 sides

$\Rightarrow x - y = 4$ ... [Dividing both sides by 4

$\Rightarrow x = 4 + y$ ...(ii)

Putting the value of x in equation (i),

$(4 + y)^2 + y2^2 = 400$

$\Rightarrow y^2 + 8y + 16 + y^2 - 400 = 0$

$\Rightarrow 2y^2 + 8y - 384 = 0$

$\Rightarrow y^2 + 4y - 192 = 0$ ... [Dividing both sides by 2

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y + 16) - 12(y + 16) = 0$

$\Rightarrow (y - 12)(y + 16) = 0$

$\Rightarrow y - 12 = 0$ or $y + 16 = 0$

$\Rightarrow y = 12$ or $y = -16$ ... [Neglecting negative value

∴ Side of small square = y = 12 cm

and Side of large square = x = 4 + 12 = 16 cm

Q.The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by 1/15. Find the fraction. (20120D)

Solution

Let the denominator be x and the numerator be x − 3.

$\therefore$ Fraction = $\frac{x-3}{x}$

New denominator = x + 1

According to the Question,

$$\Rightarrow \frac{x-3}{x+1} = \frac{x-3}{x} - \frac{1}{15}$$

$$\Rightarrow \frac{x-3}{x+1} = \frac{15x-45-x}{15x}$$

$$\Rightarrow \frac{x-3}{x+1} = \frac{14x-45}{15x}$$

$\Rightarrow 15x^2 - 45x = 14x^2 - 45x + 14x - 45$

$\Rightarrow 15x^2 - 14x^2 - 14x + 45 = 0$

Solution

$\Rightarrow x^2 - 14x + 45 = 0$

$\Rightarrow x^2 - 5x - 9x + 45 = 0$

$\Rightarrow x(x - 5) - 9(x - 5) = 0$

$\Rightarrow (x - 5)(x - 9) = 0$

$\Rightarrow x - 5 = 0$ or $x - 9 = 0$

$\Rightarrow x = 5$ or $x = 9$

When $x = 5$, fraction $= \frac{5-3}{5} = \frac{2}{5}$

When $x = 9$, fraction $= \frac{9-3}{9} = \frac{6}{9} = \frac{2}{3}$

$\therefore$ Fraction $= \frac{2}{5}$ or $\frac{2}{3}$

Q.A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park. (2016OD)

Solution

Let length of the rectangular park = x m,

breadth of the rectangular park = (x -3)m

∴ Area of the rectangular park = $x(x - 3)m^2$... (i)

Base of an isosceles triangle = (x − 3)m

Altitude of an isosceles triangle = 12 m

∴ Area of isosceles triangle

= 1/2 × base × altitude

= 1/2 × (x − 3) × 12

= 6(x − 3) ...(ii)

According to the question,

Ar.(rectangle) − Ar.(isosceles Δ) = $4\ m^2$

⇒ x(x − 3) − 6(x − 3) = 4 ... [From (i) & (ii)

⇒ $x^2 - 3x - 6x + 18 - 4 = 0$

Solution

$\Rightarrow x^2 - 9x + 14 = 0$

$\Rightarrow x^2 - 7x - 2x + 14 = 0$

$\Rightarrow x(x - 7) - 2(x - 7) = 0$

$\Rightarrow (x - 2)(x - 7) = 0$

$\Rightarrow x - 2 = 0$ or $x - 7 = 0$

$\Rightarrow x = 2$ or $x = 7$

When $x = 2$, breadth of rectangle becomes -ve, so this is not possible.

$\therefore$ Length of the rectangular park, $x = 7$ m

and Breadth $= (x - 3) = 4$ m.

Arithmetic Progressions

Q. If the sum of first m terms of an AP is the same as the sum of its first n terms, show that the sum of its first (m + n) terms is zero.

[Board 2012]

Solution

Let a be the first term and d be the common difference of the given AP. Then,

$$S_m = S_n$$

$$\frac{m}{2}\{2a+(m-1)d\} = \frac{n}{2}\{2a+(n-1)d\}$$

$$2a(m-n)+\{m(m-1)-n(n-1)d\} = 0$$

$$2a(m-n)+[(m^2-n^2)-(m-n)d] = 0$$

$$(m-n)[2a+(m+n-1)d] = 0$$

$$2a+(m+n-1)d = 0$$

Now, $$S_{m+n} = \frac{m+n}{2}\{2a+(m+n-1)d\}$$

$$= \frac{m+n}{2}\times 0 = 0$$

Q. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

[Board 2019]

Solution

Given AP is 3, 15, 27, 39......

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a + (n-1)d$$

$$a_{21} = 3 + (21-1) \times 12$$

$$= 3 + 20 \times 12 = 243$$

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243 + 120 = 363$

Therefore,

$$a_n = a + (n-1)d$$

$$363 = 3 + (n-1)12$$

$$360 = 12(n-1)$$

$$n - 1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

Q. Write the nth term of the AP

$$\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots\ldots$$

[Board 2017]

Solution

Let the first term be a, common difference be d and nth term be a_n.

We have
$$a = \frac{1}{m}$$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence ,
$$a_n = \frac{1}{m} + n - 1$$

Q. Find, if 100 is a term of the AP 25, , 28 31, ... or not.

[Board 2012]

Solution

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 100$

Here $a = 25, d = 28 - 25 = 31 - 28 = 3$

Now

$$\begin{aligned} a_n &= a + (n-1)d, \\ 100 &= 25 + (n-1) \times 3 \\ 100 - 25 = 75 &= (n-1) \times 3 \\ 25 &= n - 1 \\ n &= 26 \end{aligned}$$

Since 26 is an whole number, thus 100 is a term of given AP.

Q. If the sum of the series
2 + 5 + 8 + 11......... is 60100, then the number of terms are

[Board 2014]

Solution

We have $a = 2$. $d = 5 - 2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a + (n-1)d] = S_n$$

$$\frac{n}{2}[4 + (n-1)3] = 60100$$

$$n(3n+1) = 120200$$

$$3n^2 + n - 120200 = 0$$

$$(n-200)(3n+601) = 0 \Rightarrow n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Q. Find the middle term of the AP 213, 205, 197, ... 37.

[Board 2015]

Solution

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 213, d = 205 - 213 = -8, a_m = 37$

$$\begin{aligned} a_m &= a + (m-1)d \\ 37 &= 213 + (m-1)(-8) \\ 37 - 213 &= -8(m-1) \\ m - 1 &= \frac{-176}{-8} = 22 \\ m &= 22 + 1 = 23 \end{aligned}$$

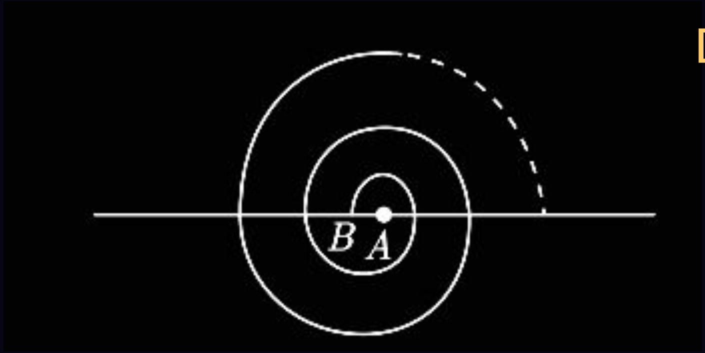
The middle term will be $= \dfrac{23+1}{2} = 12^{th}$

$$a_{12} = a + (12-1)d = 213 + (12-1)(-8)$$
$$= 213 - 88 = 125$$

Middle term will be 125.

Q. A spiral is made up of successive semicircles with centres alternately A and B starting with A, of radii 1 cm, 2 cm, 3 cm, ... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles?
(Use $\pi = 3.14$)

[Board 2012]



Solution

$$l_1 = \pi r_1 = \pi(1) = \pi \text{ cm}$$

$$l_2 = \pi r_2 = \pi(2) = 2\pi \text{ cm}$$

$$l_3 = 3\pi \text{ cm}$$

$$l_{11} = 11\pi \text{ cm}$$

$\therefore$ Total length of spiral

$$= l_1 + l_2 + \dots\dots\dots\dots + l_{11}$$

$$= \pi + 2\pi + 3\pi + \dots\dots\dots\dots + 11\pi$$

$$= \pi(1 + 2 + 3 + \dots\dots\dots + 11)$$

$$= \pi \times \frac{11 \times 12}{2}$$

$$= 66 \times 3.14$$

$$= 207.24 \text{ cm.}$$

Q. If seven times the 7th term of an AP is equal to eleven times the 11th term, then what will be its 18th term.

[Board 2017]

Solution

Let the first term be a, common difference be d and nth term be a_n.

$$7a_7 = 11a_{11}$$

Now $$7(a+6d) = 11(a+10d)$$

$$7a+42d = 11a+110d$$

$$11a-7a = 42d-110d$$

, $$4a = -68d$$

$$4a+68d = 0$$

$$4(a+17d) = 0$$

$$a+17d = 0$$

Hence, $$a_{18} = 0$$

Coordinate Geometry

Q. Find a relation between x and y such that the point P(x, y) is equidistant from the points A(-5, 3) and B(7, 2).

[Board 2016]

Solution

Let $P(x, y)$ is equidistant from $A(-5, 3)$ and $B(7, 2)$, then we have

$$\begin{aligned} AP &= BP \\ \sqrt{(x+5)^2+(y-3)^2} &= \sqrt{(x-7)^2+(y-2)^2} \\ (x+5)^2+(y-3)^2 &= (x-7)^2+(y-2)^2 \\ 10x+25-6y+9 &= -14x+49-4y+4 \\ 24x+34 &= 2y+53 \\ 24x-2y &= 19 \end{aligned}$$

Thus $24x-2y-19 = 0$ is the required relation.

Q. The x -coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) , and R(-3, 6), find the coordinates of P.

[Board 2016]

Solution

Let the coordinates of P be (x,y). It is given that x=2y

Also,

PQ=PR

$$\Rightarrow \sqrt{(x-2)^2+(y+5)^2}=\sqrt{(x+3)^2+(y-6)^2}$$

$$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2}=\sqrt{(2y+3)^2+(y-6)^2}$$

$$\Rightarrow \sqrt{5y^2+2y+29}=\sqrt{5y^2+45}$$

$$\Rightarrow 5y^2+2y+29=5y^2+45$$

$$\Rightarrow 2y=16$$

$$\Rightarrow y=8$$

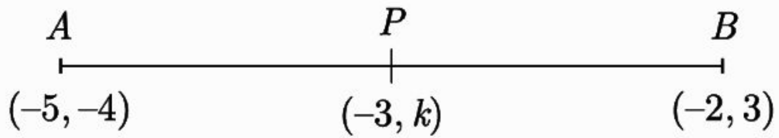
Hence, the coordinates of P are (16, 8).

Q. Find the ratio in which the point (-3, k) divides the line segment joining the points (-5, -4)and (-2, 3). Also find the value of k .

[Board 2018]

Solution

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.

x co-ordinate for section formula

$$-3 = \frac{(-2)n + 1(-5)}{n+1}$$

$$-3(n+1) = -2n - 5$$

$$-3n - 3 = -2n - 5$$

$$5 - 3 = 3n - 2n$$

$$2 = n$$

Ratio $\frac{n}{1} = \frac{2}{1}$ or $2:1$

Now, y co-ordinate,

$$k = \frac{2(3) + 1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

Q. Prove that the point (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

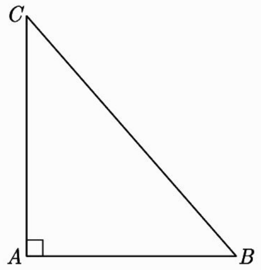
[Board 2016]

Solution

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now
$$\begin{aligned} AB^2 &= (3-6)^2 + (0-4)^2 \\ &= 9 + 16 = 25 \\ BC^2 &= (6+1)^2 + (4-3)^2 \\ &= 49 + 1 = 50 \\ CA^2 &= (-1-3)^2 + (3-0)^2 \\ &= 16 + 9 = 25 \\ AB^2 &= CA^2 \text{ or, } AB = CA \end{aligned}$$

Hence triangle is isosceles.



Also, $\quad 25 + 25 = 50$

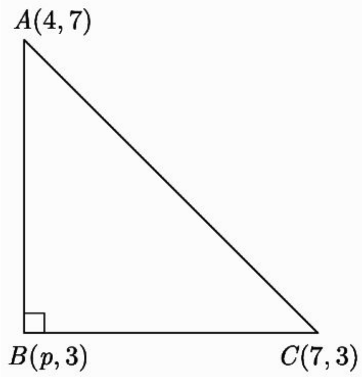
or, $\quad AB^2 + CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

Q. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B . Find the value of p.

[Board 2015]

Solution



$$AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2$$

$$= (7-4)^2 + (3-4)^2$$

$$(p-4)^2 + (-4)^2 + (7-p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

$$(p-4)(p-7) = 0$$

$$p = 7 \text{ or } 4$$

Q. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the point $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

[Board 2012]

Solution

Let point P divide AB in the ratio 1 : k

Then, by section formula,

$$\left(\frac{3}{4}, \frac{5}{12}\right) \equiv \left(\frac{k\left(\frac{1}{2}\right) + 1(2)}{k+1}, \frac{k\left(\frac{3}{2}\right) + 1(-5)}{k+1}\right)$$

Equating x and y coordinates, we get

$$\frac{k/2 + 2}{k+1} = \frac{3}{4}, \frac{3k/2 - 5}{k+1} = \frac{5}{12}$$

$$\Rightarrow 2k + 8 = 3k + 3, 18k - 60 = 5k + 5$$

$$\Rightarrow k = 5$$

Hence P divides AB in the ratio 1 : 5

Q. In what ratio does the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8)?

[CBSE 2017]

Solution

Let $AP : PB = k : 1$

Now $\dfrac{3k-6}{k+1} = -4$

$$3k - 6 = -4k - 4$$

$$7k = 2 \Rightarrow k = \frac{2}{7}$$

Hence, $AP : PB = 2 : 7$

Q. Show that the points (a, a), (-a, -a) and ($-\sqrt{3}a$, $\sqrt{3}a$) are the vertices of an equilateral triangle.

[Board 2015]

Solution

Let $A(a,a), B(-a,-a)$ and $C(-\sqrt{3}a, \sqrt{3}a)$.

Now
$$AB = \sqrt{(a+a)^2+(a+a)^2}$$
$$= \sqrt{4a^2+4a^2} = 2\sqrt{2}a$$
$$BC = \sqrt{(-a+\sqrt{3}a)^2+(-a-\sqrt{3}a)^2}$$
$$= \sqrt{a^2-2\sqrt{3}a^2+3a^2+a^2+2\sqrt{3}a^2+3a^2}$$
$$= 2\sqrt{2}a$$
$$AC = \sqrt{(a+\sqrt{3}a)^2+(a-\sqrt{3}a)^2}$$
$$= \sqrt{a^2+2\sqrt{3}a^2+3a^2+a^2-2\sqrt{3}a^2+3a^2}$$
$$= 2\sqrt{2}a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

Q. Show that A(-1, 0), B(3, 1), C(2, 2) and D(-2, 1) are the vertices of a parallelogram ABCD.

[Board 2012]

Solution

Mid-point of AC,

$$\left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Mid-point of BD,

$$\left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

Q. If P(2, -1), Q(3, 4), R(-2, 3) and S(- 3, -2) be four points in a plane, show that PQRS is a rhombus but not a square.

[CBSE 2012]

Solution

We have $P(2, -1)$, $Q(3, 4)$, $R(-2, 3)$, $S(-3, -2)$

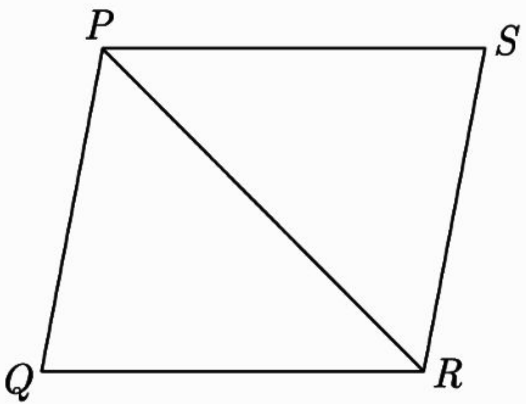
$$PQ = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$QR = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$RS = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$PS = \sqrt{5^2 + 1^2} = \sqrt{26}$$

Since all the four sides are equal, $PQRS$ is a rhombus.



Solution

Now $$PR = \sqrt{1^2 + 5^2} = \sqrt{26}$$
$$= \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$PQ^2 + QR^2 = 2 \times 26 = 52 \neq (\sqrt{32})^2$$

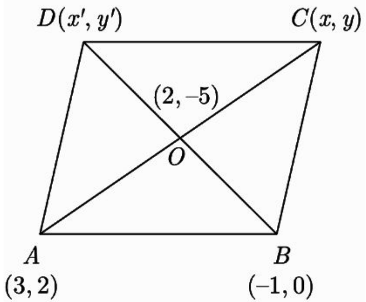
Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

Q. If two adjacent vertices of a parallelogram are (3, 2) and (-1, 0) and the diagonals intersect at (2, -5), then find the coordinates of the other two vertices.

[Board 2017]

Solution

Let two other co-ordinates be (x, y) and (x', y') respectively using mid-point formula.
As per question parallelogram is shown below.



Now $\quad 2 = \dfrac{x+3}{2} \Rightarrow x = 1$

and $\quad -5 = \dfrac{2+y}{2} \Rightarrow y = -12$

Again, $\quad \dfrac{-1+x'}{2} = 2 \Rightarrow x' = 5$

and $\quad \dfrac{0+y'}{2} = -5 \Rightarrow y' = -10$

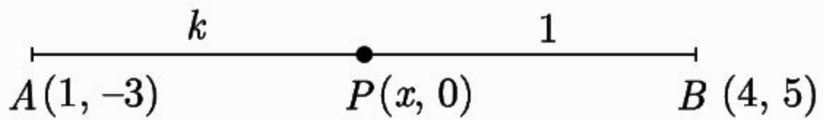
Hence, coordinates of $C(1, -12)$ and $D(5, -10)$

Q. Find the ratio in which the segment joining the points (1, -3) and (4, 5) is divided by x -axis? Also find the coordinates of this point on x -axis.

[Board 2019]

Solution

Let the required ratio be $k:1$ and the point on x-axis be $(x, 0)$.



Here, $(x_1, y_1) = (1, -3)$

and $(x_2, y_2) = (4, 5)$

Using section formula y coordinate, we obtain,

$$y = \frac{my_2 + ny_1}{m+n}$$

$$0 = \frac{k \times 5 + 1 \times 1(-3)}{k+1}$$

$$0 = 5k - 3$$

$$5k = 3 \Rightarrow k = \frac{3}{5}$$

Solution

Hence, the required ratio is $\frac{3}{5}$ i.e $3:5$.
Now, again using section formula for x, we obtain

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{k \times (4) + 1 \times 1}{k+1}$$

$$= \frac{\frac{3}{5}(4)+1}{\frac{3}{5}+1} = \frac{12+5}{3+5} = \frac{17}{8}$$

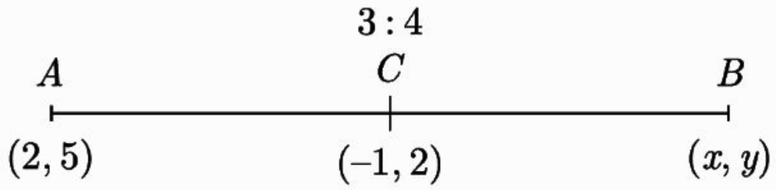
Co-ordinate of P is $\left(\frac{17}{8}, 0\right)$.

Q. If the point C(-1, 2), divides internally the line segment joining the points A(2, 5) and B(x, y) in the ratio 3 : 4, find the value of $x^2 + y^2$.

[Board 2016]

Solution

As per question, line diagram is shown below.



We have $\frac{AC}{BC} = \frac{3}{4}$

Applying section formula for x co-ordinate,

$$-1 = \frac{3x+4(2)}{3+4}$$

$$-7 = 3x+8 \Rightarrow x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y+4(5)}{3+4}$$

$$14 = 3y+20 \Rightarrow y = -2$$

Solution

Thus (x, y) is $(-5, -2)$.

Now

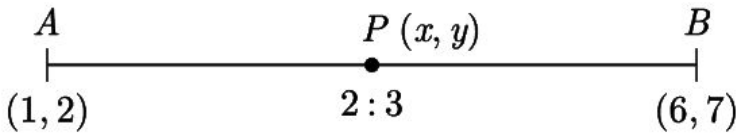
$$\begin{aligned} x^2 + y^2 &= (-5)^2 + (-2)^2 \\ &= 25 + 4 = 29 \end{aligned}$$

Q. Find the coordinates of a point P on the line segment joining A(1, 2) and B(6, 7) such that $AP = \frac{2}{5} AB$.

[Board 2015]

Solution

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP\!:\!PB = 2\!:\!3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{14+6}{5} = 4$$

Thus $P(x, y) = (3, 4)$

Q. Find the coordinates of the points of trisection of the line segment joining the points A(1, -2) and B(-3, 4).

[Board 2012]

Solution

Let $P(x_1, y_1), Q(x_2, y_2)$ divides AB into 3 equal parts.
Thus P divides AB in the ratio of 1:2.
As per question, line diagram is shown below.

Now $$x_1 = \frac{1(-3) + 2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4) + 2(-2)}{1+2} = \frac{4-4}{3} = 0$$

Co-ordinates of P is $\left(-\frac{1}{3}, 0\right)$.

Solution

Co-ordinates of P is $(-\frac{1}{3}, 0)$.

Here Q is mid-point of PB.

Thus
$$x_2 = \frac{-\frac{1}{3} + (-3)}{2}$$
$$= \frac{-10}{6} = \frac{-5}{3}$$
$$y_2 = \frac{0+4}{2} = 2$$

Thus co-ordinates of Q is $(-\frac{5}{2}, 2)$.

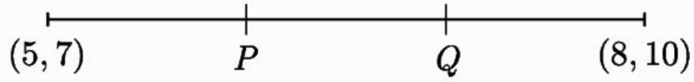
Q. Find the coordinates of the points which divide the line segment joining the points (5, 7) and (8, 10) in 3 equal parts.

[Board 2017]

Solution

Let $P(x_1, y_2)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.



Using section formula we have,

Now
$$x = \frac{1(8) + 2(5)}{3} = 6$$

$$y = \frac{1(10) + 2(7)}{3} = 8$$

Thus $P(x_1, y_1)$ is $P(6,8)$. Since Q is the mid point of PB, we have

$$x_1 = \frac{6+8}{2} = 7$$

$$y_1 = \frac{8+10}{2} = 9$$

Thus $Q(x_2, y_2)$ is $Q(7,9)$

Q. If (a, b) is the mid-point of the segment joining the points A(10, -6) and B(k, 4) and $a - 2b = 18$, find the value of k and the distance AB .

[Board 2012]

Solution

We have $A(10,-6)$ and $B(k,4)$.
If $P(a,b)$ is mid-point of AB, then we have

$$(a,b) = \left(\frac{k+10}{2}, \frac{-6+4}{2}\right)$$

$$a = \frac{k+10}{2} \text{ and } b = -1$$

From given condition we have

$$a-2b = 18$$

Substituting value $b=-1$ we obtain

$$a+2 = 18 \Rightarrow a = 16$$

$$a = \frac{k+10}{2} = 16 \Rightarrow k = 22$$

$$P(a,b) = (16,1)$$

$$AB = \sqrt{(22-10)^2+(4+6)^2}$$

$$= 2\sqrt{61} \text{ units}$$

Q. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$.

[Board 2016]

Solution

Since $P(x, y)$ is equidistant from the given points $A(5,1)$ and $B(-1,5)$,

$$PA = PB$$

$$PA^2 = PB^2$$

Using distance formula,

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$

$$(5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$$

$$25 - 10x + 1 - 2y = 1 + 2x + 25 - 10y$$

$$-10x - 2y = 2x - 10y$$

$$8y = 12x$$

$$3x = 2y$$

Hence proved.

Q. The base QR of an equilateral triangle PQR lies on x-axis. The coordinates of the point Q are (-4, 0) and origin is the midpoint of the base. Find the coordinates of the points P and R.

[Board 2018]

Solution

Let (x, 0) be the coordinates of R. Then

$$O = \frac{-4+x}{2} \Rightarrow x = 4$$

Thus, the coordinates of R are (4, 0).

Here, PQ = QR = PR and the coordinates of P lies on y-axis. Let the coordinates of P be (O, y). Then

PQ = QR

$\Rightarrow PQ^2 = QR^2$

$\Rightarrow (O + 4)^2 + (y - O)^2 = 8^2$

$\Rightarrow y^2 = 64 - 16 = 48$

$$\Rightarrow y = \pm 4\sqrt{3}$$

Hence, the required coordinates are

$$R\,(4,0) \text{ and } P\left(O, 4\sqrt{3}\right) \text{ or } P\left(O, -4\sqrt{3}\right)$$

Q. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

[Board 2019]

Solution

If the sum of the lengths of any two line segments is equal to the length of the third line segment then all three points are collinear.

Consider, A = (1, 5), B = (2, 3) and C = (-2, -11)

Find the distance between points; say AB, BC and CA

$$AB = \sqrt{(2-1)^2 + (3-5)^2}$$
$$= \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$
$$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$$
$$= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$$
$$CA = \sqrt{(-2-1)^2 + (-11-5)^2}$$
$$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

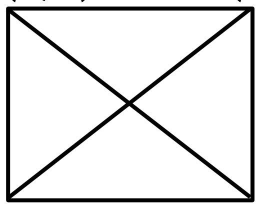
Since $\mathrm{AB} + \mathrm{BC} \neq \mathrm{CA}$

Therefore, the points $(1, 5)$, $(2, 3)$, and $(-2, -11)$ are not collinear.

Q. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.

Solution

Let the given points are A(3, 0), B(4, 5), C(-1, 4) and D(-2, -1).



$$BD = \sqrt{(-2-4)^2 + (-1-5)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72}$$

$$AC = \sqrt{(-1-3)^2 + (4-0)^2}$$

$$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$$

Now, area of rhombus ABCD

$$= \frac{1}{2} \times (\text{Product of diagonals})$$

$$\Rightarrow \text{Area of rhombus} = \frac{1}{2} \times (AC \times BD)$$

$$= \left[\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}\right] \text{ sq. units}$$

$$= \left(2\sqrt{2} \times 6\sqrt{2}\right) \text{ sq. units} = 24 \text{ sq. units.}$$

Q. The three vertices of a parallelogram ABCD are A(3, -4), B(-1, -3) and C(-6, 2). Find the coordinates of vertex D.

Solution

The three vertices of the parallelogram ABCD are A(3, -4), B(-1, -3) and C(-6, 2).

Let the coordinates of the vertex D be (x, y).

It is known that in a parallelogram, the diagonals bisect each other.

∴ Midpoint of AC = Midpoint of BD

$$\Rightarrow\left(\frac{3-6}{2},\frac{-4+2}{2}\right)=\left(\frac{-1+x}{2},\frac{-3+y}{2}\right)$$

$$\Rightarrow\left(-\frac{3}{2},-\frac{2}{2}\right)=\left(\frac{-1+x}{2},\frac{-3+y}{2}\right)$$

$$\Rightarrow-\frac{3}{2}=\frac{-1+x}{2},-\frac{2}{2}=\frac{-3+y}{2}$$

$$\Rightarrow x=-2,\ y=1$$

So, the coordinates of the vertex D is $(-2,1)$.

Q. Find a relation between x and y such that the point P(x, y) is equidistant from the points A(-5, 3) and B(7, 2).

[Board 2016]

Solution

Let $P(x, y)$ is equidistant from $A(-5, 3)$ and $B(7, 2)$, then we have

$$\begin{aligned} AP &= BP \\ \sqrt{(x+5)^2+(y-3)^2} &= \sqrt{(x-7)^2+(y-2)^2} \\ (x+5)^2+(y-3)^2 &= (x-7)^2+(y-2)^2 \\ 10x+25-6y+9 &= -14x+49-4y+4 \\ 24x+34 &= 2y+53 \\ 24x-2y &= 19 \end{aligned}$$

Thus $24x-2y-19 = 0$ is the required relation.

Q. The x -coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) , and R(-3, 6), find the coordinates of P.

[Board 2016]

Solution

Let the coordinates of P be (x,y). It is given that x=2y

Also,

PQ=PR

$$\Rightarrow \sqrt{(x-2)^2+(y+5)^2} = \sqrt{(x+3)^2+(y-6)^2}$$

$$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$$\Rightarrow \sqrt{5y^2+2y+29} = \sqrt{5y^2+45}$$

$$\Rightarrow 5y^2+2y+29 = 5y^2+45$$

$$\Rightarrow 2y = 16$$

$$\Rightarrow y = 8$$

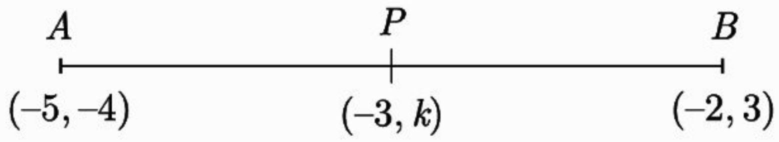
Hence, the coordinates of P are (16, 8).

Q. Find the ratio in which the point (-3, k) divides the line segment joining the points (-5, -4)and (-2, 3). Also find the value of k .

[Board 2018]

Solution

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.

x co-ordinate for section formula

$$-3 = \frac{(-2)n + 1(-5)}{n+1}$$

$$-3(n+1) = -2n - 5$$

$$-3n - 3 = -2n - 5$$

$$5 - 3 = 3n - 2n$$

$$2 = n$$

Ratio $\quad \frac{n}{1} = \frac{2}{1}$ or $2:1$

Now, y co-ordinate,

$$k = \frac{2(3) + 1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

Q. Prove that the point (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

[Board 2016]

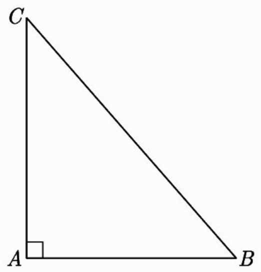
Solution

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now

$$\begin{aligned} AB^2 &= (3-6)^2 + (0-4)^2 \\ &= 9 + 16 = 25 \\ BC^2 &= (6+1)^2 + (4-3)^2 \\ &= 49 + 1 = 50 \\ CA^2 &= (-1-3)^2 + (3-0)^2 \\ &= 16 + 9 = 25 \\ AB^2 &= CA^2 \text{ or, } AB = CA \end{aligned}$$

Hence triangle is isosceles.



Also, $25 + 25 = 50$

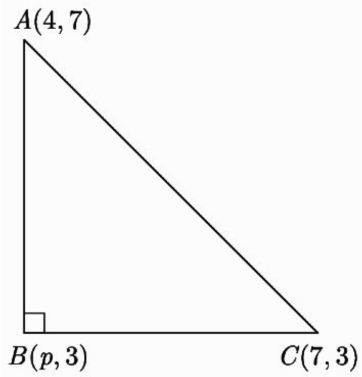
or, $AB^2 + CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

Q. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B . Find the value of p.

[Board 2015]

Solution



$$AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2 = (7-4)^2 + (3-4)^2$$

$$(p-4)^2 + (-4)^2 + (7-p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

$$(p-4)(p-7) = 0$$

$$p = 7 \text{ or } 4$$

Q. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the point $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

[Board 2012]

Solution

Let point P divide AB in the ratio 1 : k

Then, by section formula,

$$\left(\frac{3}{4}, \frac{5}{12}\right) \equiv \left(\frac{k\left(\frac{1}{2}\right) + 1(2)}{k+1}, \frac{k\left(\frac{3}{2}\right) + 1(-5)}{k+1}\right)$$

Equating x and y coordinates, we get

$$\frac{k/2 + 2}{k+1} = \frac{3}{4}, \frac{3k/2 - 5}{k+1} = \frac{5}{12}$$

$$\Rightarrow 2k + 8 = 3k + 3, 18k - 60 = 5k + 5$$

$$\Rightarrow k = 5$$

Hence P divides AB in the ratio 1 : 5

Q. In what ratio does the point P(-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8)?

[CBSE 2017]

Solution

Let $AP : PB = k : 1$

Now $\dfrac{3k-6}{k+1} = -4$

$3k - 6 = -4k - 4$

$7k = 2 \Rightarrow k = \dfrac{2}{7}$

Hence, $AP : PB = 2 : 7$

Q. Show that the points (a, a), (-a, -a) and ($-\sqrt{3}a$, $\sqrt{3}a$) are the vertices of an equilateral triangle.

[Board 2015]

Solution

Let $A(a, a), B(-a, -a)$ and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$.

Now $$AB = \sqrt{(a+a)^2 + (a+a)^2}$$

$$= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\,a$$

$$BC = \sqrt{(-a+\sqrt{3}\,a)^2 + (-a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 - 2\sqrt{3}\,a^2 + 3a^2 + a^2 + 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

$$AC = \sqrt{(a+\sqrt{3}\,a)^2 + (a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 + 2\sqrt{3}\,a^2 + 3a^2 + a^2 - 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

Q. Show that A(-1, 0), B(3, 1), C(2, 2) and D(-2, 1) are the vertices of a parallelogram ABCD.

[Board 2012]

Solution

Mid-point of AC,

$$\left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Mid-point of BD,

$$\left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

Q. If P(2, -1), Q(3, 4), R(-2, 3) and S(- 3, -2) be four points in a plane, show that PQRS is a rhombus but not a square.

[CBSE 2012]

Solution

We have $P(2, -1)$, $Q(3, 4)$, $R(-2, 3)$, $S(-3, -2)$

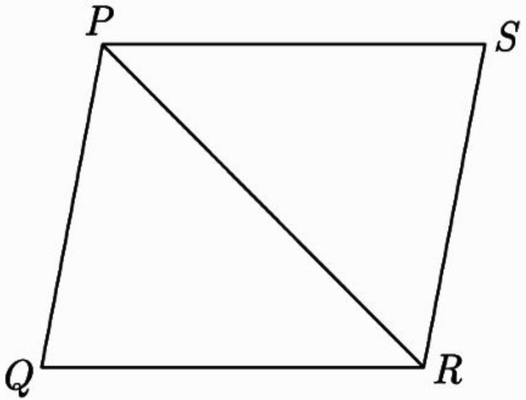
$$PQ = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$QR = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$RS = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$PS = \sqrt{5^2 + 1^2} = \sqrt{26}$$

Since all the four sides are equal, $PQRS$ is a rhombus.



Solution

Now
$$PR = \sqrt{1^2 + 5^2} = \sqrt{26}$$
$$= \sqrt{4^2 + 4^2} = \sqrt{32}$$
$$PQ^2 + QR^2 = 2 \times 26 = 52 \neq (\sqrt{32})^2$$

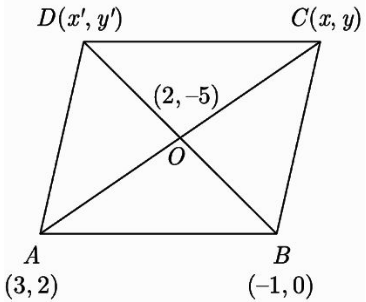
Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

Q. If two adjacent vertices of a parallelogram are (3, 2) and (-1, 0) and the diagonals intersect at (2, -5), then find the coordinates of the other two vertices.

[Board 2017]

Solution

Let two other co-ordinates be (x, y) and (x', y') respectively using mid-point formula.
As per question parallelogram is shown below.



Now $$2 = \frac{x+3}{2} \Rightarrow x = 1$$

and $$-5 = \frac{2+y}{2} \Rightarrow y = -12$$

Again, $$\frac{-1+x'}{2} = 2 \Rightarrow x' = 5$$

and $$\frac{0+y'}{2} = -5 \Rightarrow y' = -10$$

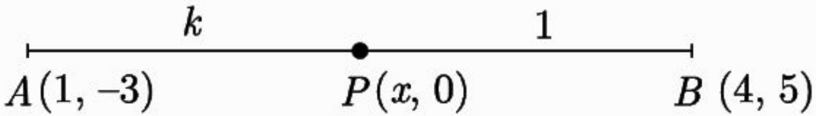
Hence, coordinates of $C(1, -12)$ and $D(5, -10)$

Q. Find the ratio in which the segment joining the points (1, -3) and (4, 5) is divided by x -axis? Also find the coordinates of this point on x -axis.

[Board 2019]

Solution

Let the required ratio be $k:1$ and the point on x-axis be $(x, 0)$.



Here, $(x_1, y_1) = (1, -3)$

and $(x_2, y_2) = (4, 5)$

Using section formula y coordinate, we obtain,

$$y = \frac{my_2 + ny_1}{m + n}$$

$$0 = \frac{k \times 5 + 1 \times 1(-3)}{k + 1}$$

$$0 = 5k - 3$$

$$5k = 3 \Rightarrow k = \frac{3}{5}$$

Solution

Hence, the required ratio is $\frac{3}{5}$ i.e $3:5$.

Now, again using section formula for x, we obtain

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{k \times (4) + 1 \times 1}{k+1}$$

$$= \frac{\frac{3}{5}(4)+1}{\frac{3}{5}+1} = \frac{12+5}{3+5} = \frac{17}{8}$$

Co-ordinate of P is $\left(\frac{17}{8}, 0\right)$.

Geometry-Triangle

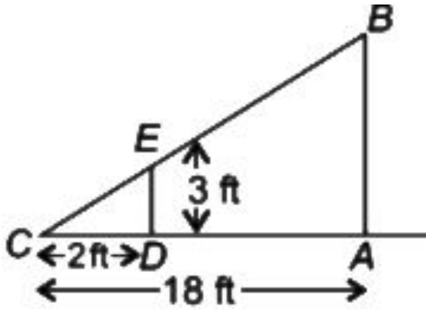
Q. If a tree casts a 18 feet shadow and at the same time, a child of height 3 feet casts a 2 feet shadow then the height of the tree is

Solution

Let AB be the height of free which casts a shadow AC = 18 feet and ED be the height of child which casts a shadow

CD = 2 feet.

Now, in ΔABC and ΔDEC,



we have

∠A = ∠D [Each 90°]

∠C = ∠C [Common angle]

∴ ΔABC ~ ΔDEC [By AA similarity]

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DC}$$

$$\Rightarrow \frac{AB}{3} = \frac{18}{2} \Rightarrow AB = 27 \text{ feet}$$

So, the height of tree is 27 feet.

Q. Given that $\Delta ABC \sim \Delta DEF$. If DE = 2AB and BC = 6 cm then, EF is equal to

Solution

$\Delta ABC \sim \Delta DEF$ (given)

$2AB = DE,\ BC = 6cm$ (given)

$\angle E = \angle B$ and $\angle D = \angle A$ and $\angle F = \angle C$

$2AB = DE$

$$\Rightarrow \frac{AB}{DE} = \frac{1}{2}$$

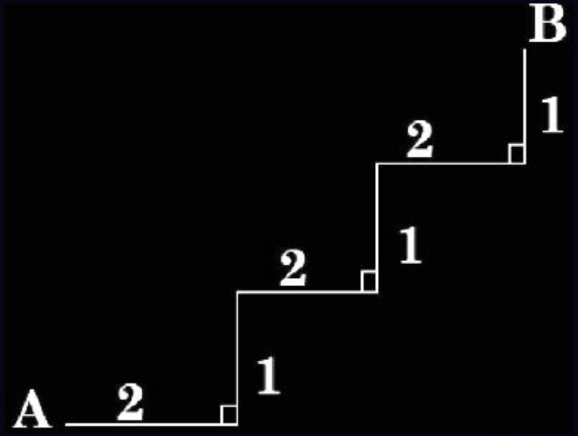
Therefore,

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{1}{2} = \frac{6}{EF}$$

or $EF = 12cm$

Q. The straight line distance between A and B is ___ units.



Solution

Then by Pythagorean theorem we have

$$hypotenuse = \sqrt{leg^2 + leg^2}$$

$$= \sqrt{2^2 + 1^2}$$

$$= \sqrt{5}.$$

From the figure it is clear that $length of AB = 3 \times hypotenuse$.

Hence straight line length of $AB = 3 \times \sqrt{5} = 3\sqrt{5}$

Q. In $\triangle ABC$, AB = 3 and, AC = 4 cm and AD is the bisector of $\angle A$. Then, BD : DC is —

Solution

In $\triangle ABC$,

AD bisects $\angle A$

By angle bisector theorem,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{BD}{DC} = \frac{3}{4}$$

Q. A boy walks 200 m towards East and then 150 m towards North. The distance of the boy from the starting point is:

Solution

The required distance = $\sqrt{(200^2 + 150^2)}$

= $\sqrt{(40000+ 22500)}$

= $\sqrt{(62500)}$ = 250m

So the distance of the girl from the starting point is 250m.

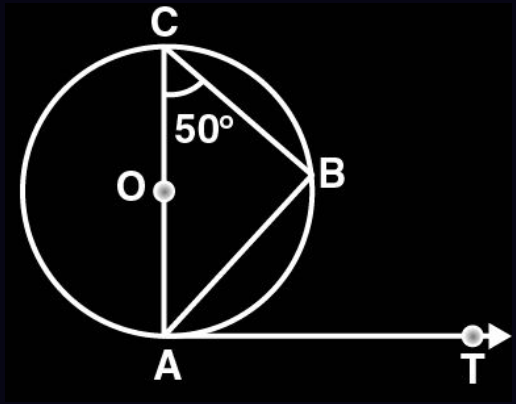
Q. If the angle between two radii of a circle is 110°, then the angle between the tangents at the ends of the radii is:

Solution

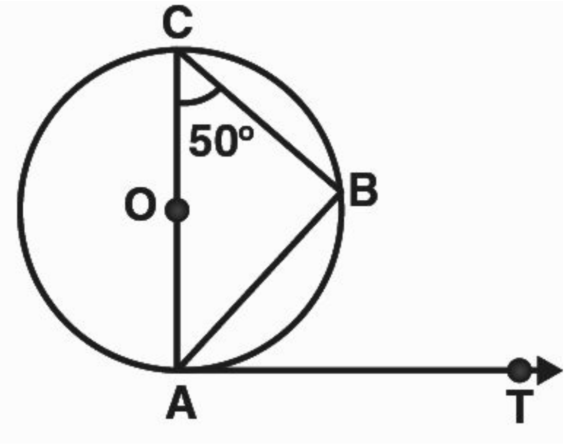
Answer: (c) 70°

Explanation: If the angle between two radii of a circle is 110°, then the angle between tangents is 180° – 110° = 70°. (By circles and tangents properties)

Q. AB is a chord of the circle and AOC is its diameter such that angle ACB = 50°. If AT is the tangent to the circle at the point A, then BAT is equal to



Solution



Answer: (c) 50°

Explanation: As per the given question:

∠ABC = 90° (angle in Semicircle is right angle)

In ΔACB

∠A + ∠B + ∠C = 180°

∠A = 180° − (90° + 50°)

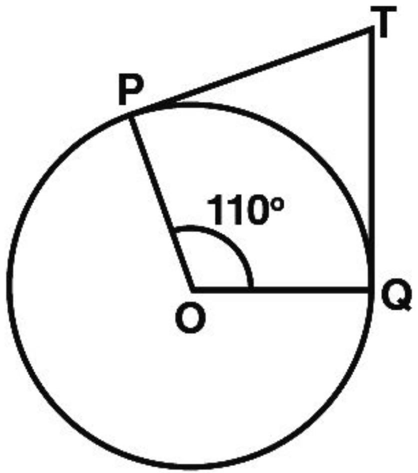
∠A = 40°

Or ∠OAB = 40°

Therefore, ∠BAT = 90° − 40° = 50°

Q. If TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

Solution



Explanation: As per the given question:

We can see, OP is the radius of the circle to the tangent PT and OQ is the radius to the tangents TQ.

So, OP $\perp$ PT and TQ $\perp$ OQ

$\therefore \angle OPT = \angle OQT = 90°$

Now, in the quadrilateral POQT, we know that the sum of the interior angles is 360°

So, $\angle PTQ + \angle POQ + \angle OPT + \angle OQT = 360°$

Now, by putting the respective values, we get,

$\Rightarrow \angle PTQ + 90° + 110° + 90° = 360°$

$\Rightarrow \angle PTQ = 70°$

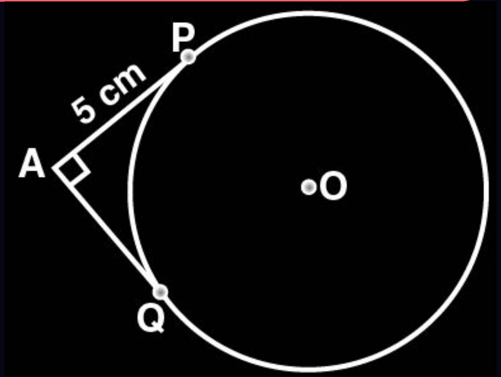
Q. If a parallelogram circumscribes a circle, then it is a:

Solution

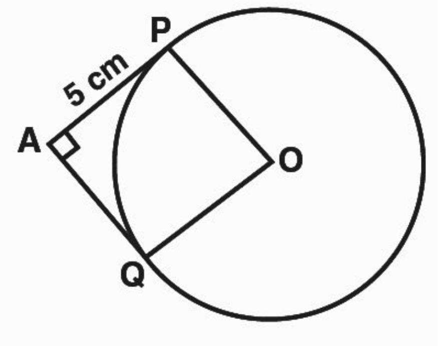
Answer: (c) Rhombus

If a parallelogram circumscribes a circle, then it is a Rhombus.

Q. In the figure below, the pair of tangents AP and AQ drawn from an external point A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. Then the radius of the circle is



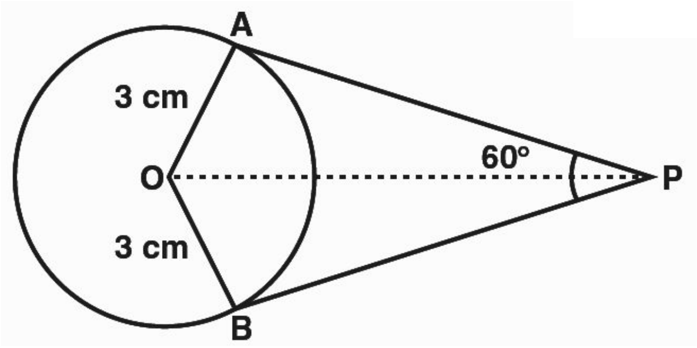
Solution



Join OP and OQ.
Tangents AP = AQ
In triangle APO and AQO,
AP = AQ
AO = AO (Common)
OP = OQ (radius of same circle)
Thus, ΔAPO ~ ΔAQO.
POQA is a square
OP = OQ = AP = AQ
So, AP = AQ = 5 Cm
And AP = OP (Proved)
Therefore, radius = OP = 5 cm

Q. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

Solution



ΔAOP and ΔBOP are congruent.
Therefore, ∠APO = ∠BPO = 60°/2 = 30°
OA is perpendicular to AP.
In right triangle AOP,
tan 30° = OA/AP
1/√3 = 3/AP
AP = 3√3
Hence, the length of the tangent is 3√3 cm.

Q. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3), then the value of x is

Solution

Since, A and B lie on the circle having centre O.

$$\begin{aligned} OA &= OB \\ \sqrt{(4-2)^2+(3-3)^2} &= \sqrt{(x-2)^2+(5-3)^2} \\ 2 &= \sqrt{(x-2)^2+4} \\ 4 &= (x-2)^2+4 \\ (x-2)^2 &= 0 \Rightarrow x = 2 \end{aligned}$$

Thus (c) is correct option.

Q. The ratio in which the point (2, y) divides the join of (- 4, 3) and (6, 3), hence the value of y is

Solution

Let the required ratio be $k:1$

Then, $$2 = \frac{6k - 4(1)}{k+1}$$

or $$k = \frac{3}{2}$$

The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $$y = \frac{3(3)+2(3)}{3+2} = 3$$

Thus (c) is correct option.

Q. The point P on x -axis equidistant from the points A(- 1, 0) and B(5, 0) is

[Board 2020 OD Standard]

Solution

Let the position of the point P on x-axis be $(x, 0)$, then

$$\begin{aligned} PA^2 &= PB^2 \\ (x+1)^2 + (0)^2 &= (5-x)^2 + (0)^2 \\ x^2 + 2x + 1 &= 25 + x^2 - 10x \\ 2x + 10x &= 25 - 1 \\ 12x &= 24 \Rightarrow x = 2 \end{aligned}$$

Hence, the point $P(x, 0)$ is (2, 0).
Thus (a) is correct option.

Alternative :

You may easily observe that both point $A(-1, 0)$ and $B(5, 0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1, 0)$ and $B(5, 0)$.

$$x = \frac{-1+5}{2} = 2$$

Q. The distance between the points $(a \cos\theta + b \sin\theta, 0)$ and $(0, a \sin\theta - b \cos\theta)$ is

[Board 2020 Delhi Standard]

Solution

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos\theta$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2$$

$$= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta +$$

$$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$

$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$

$$= a^2 \times 1 + b^2 \times 1 = a^2 + b^2$$

Thus $d^2 = a^2 + b^2$

$$d = \sqrt{a^2 + b^2}$$

Therefore (c) is correct option.

Q. If $A(m/3, 5)$ is the mid-point of the line segment joining the points $Q(-6, 7)$ and $R(-2, 3)$, then the value of m is

[Board 2020 SQP Standard]

Solution

Given points are $Q(-6,7)$ and $R(-2,3)$

Mid point $A\left(\frac{m}{3},\ 5\right) = \left(\frac{-6-2}{2},\ \frac{7+3}{2}\right)$

$= (-4, 5)$

Equating, $\frac{m}{3} = -4 \Rightarrow m = -12$

Thus (a) is correct option.

Q. If three points (0, 0), (3, $\sqrt{3}$) and (3, λ) form an equilateral triangle, then λ equals

Solution

Let the given points are $A(0,0)$, $B(3,\sqrt{3})$ and $C(3,\lambda)$.
Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\sqrt{(3-0)^2+(\sqrt{3}-0)^2} = \sqrt{(3-0)^2+(\lambda-0)^2}$$

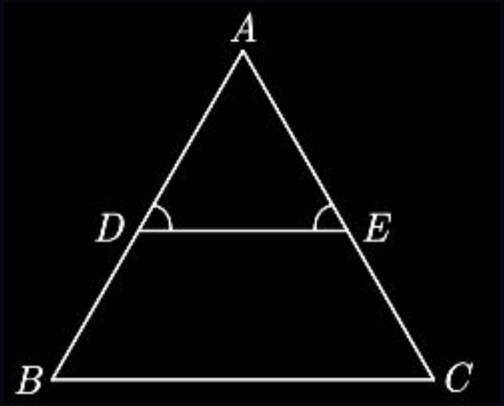
$$9+3 = 9+\lambda^2$$

$$\lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$$

Thus (d) is correct option.

Q. In Figure $\angle D = \angle E$ and $AD/DB = AE/EC$, prove that 🔺 BAC is an isosceles triangle.

[Board 2020]



Solution

We have, $\angle D = \angle E$

and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

Due to corresponding angles we have

$\angle ADE = \angle ABC$ and

$\angle AED = \angle ACB$

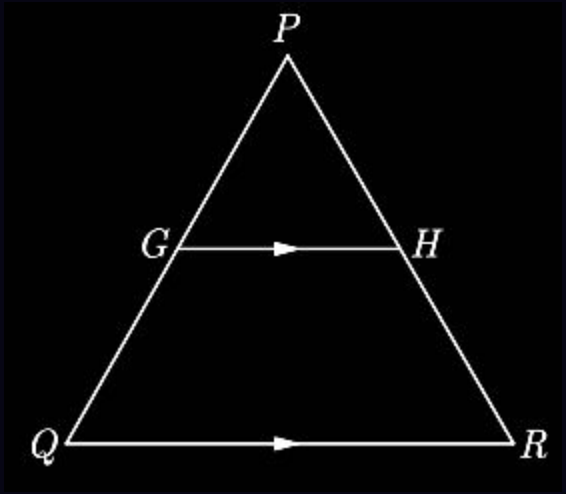
Given $\angle ADE = \angle AED$

Thus $\angle ABC = \angle ACB$

Therefore BAC is an isosceles triangle.

Q. In the given figure, G is the mid-point of the side PQ of △ PQR and GH || QR. Prove that H is the mid-point of the side PR or the triangle PQR.

[Board 2012]



Solution

Since G is the mid-point of PQ we have

$$PG = GQ$$

$$\frac{PG}{GQ} = 1$$

We also have $GH || QR$, thus by BPT we get

$$\frac{PG}{GQ} = \frac{PH}{HR}$$

$$1 = \frac{PH}{HR}$$

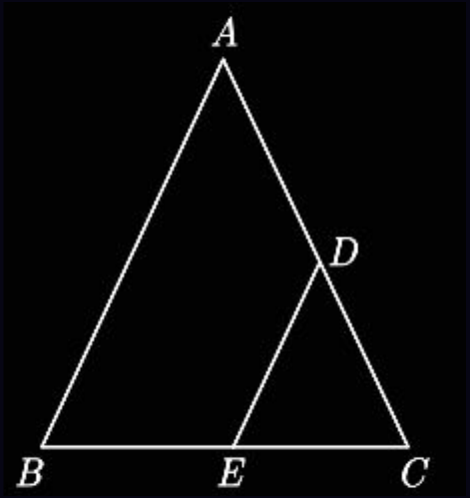
$$PH = HR.$$

Hence proved.

Hence, H is the mid-point of PR.

Q. In the figure of △ABC, the points D and E are on the sides CA, CB respectively such that DE || AB, AD = 2x, DC = x + 3, BE = 2x − 1 and CE = x. Then, find x.

[Board 2015, 16]



Solution

We have $$\frac{CD}{AD} = \frac{CE}{BE}$$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC, $DE \parallel AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CA-CD} = \frac{CE}{CB-CE}$$

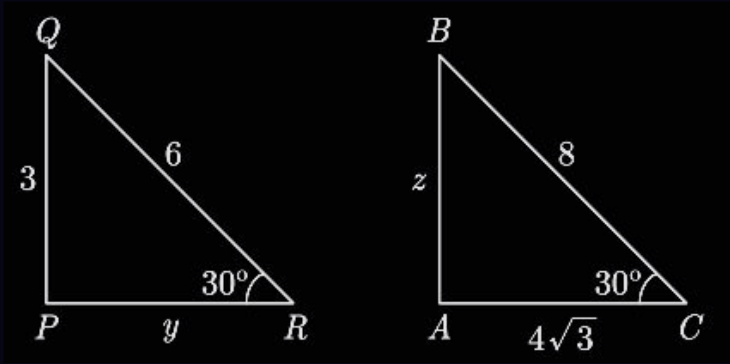
$$\frac{CD}{AD} = \frac{CE}{BE}$$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Q. In the given figure, $\triangle ABC \sim \triangle PQR$. Find the value of y + z.

[Board 2010]



Solution

In the given figure $\Delta ABC \sim \Delta PQR$,

Thus $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

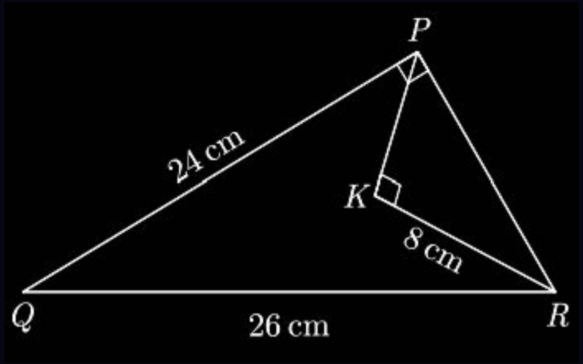
$$z = \frac{8 \times 3}{6} \text{ and } y = \frac{4\sqrt{3} \times 6}{8}$$

$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus $$y + z = 3\sqrt{3} + 4$$

Q. In the given triangle PQR, $\angle QPR = 90°$, PQ = 24 cm and QR = 26 cm and in $\triangle$ PKR, $\angle PKR = 90°$ and KR = 8 cm, find PK.

[Board 2011]



Solution

In the given triangle we have

$$\angle QPR = 90^0$$

Thus $$QR^2 = QP^2 + PR^2$$

$$PR = \sqrt{26^2 - 24^2}$$

$$= \sqrt{100} = 10 \text{ cm}$$

Now $$\angle PKR = 90^0$$

Thus $$PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$

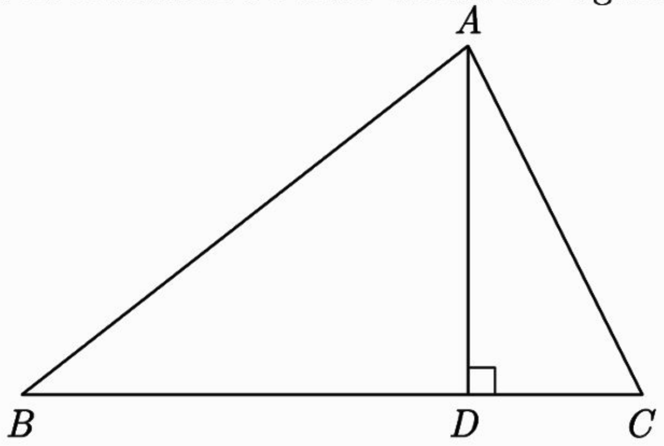
$$= \sqrt{36} = 6 \text{ cm}$$

Q. In $\triangle$ABC, AD $\perp$ BC, such that $AD^2 = BD \times CD$. Prove that $\triangle$ABC is right angled at A.

[Board 2015]

Solution

As per given condition we have drawn the figure below.



We have $AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

Since $\angle D = 90^\circ$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $\angle BAD = \angle ACD;$

Solution

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^{\circ}$$

$$2\angle BAD + 2\angle DAC = 180^{\circ}$$

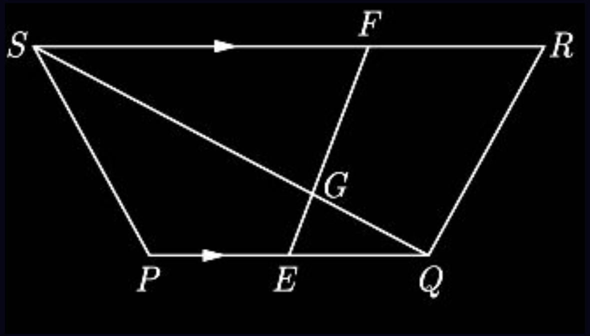
$$\angle BAD + \angle DAC = 90^{\circ}$$

$$\angle A = 90^{\circ}$$

Thus ΔABC is right angled at A.

Q. In the figure, PQRS is a trapezium in which PQ || RS. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G . Prove that
$EQ \times GS = GQ \times FS$.

[Board 2016]



Solution

In $\Delta\, GEQ$ and $\Delta\, GFS$,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

Due to alternate angle,

$$\angle EQG = \angle FSG$$

Thus by AA similarity we have

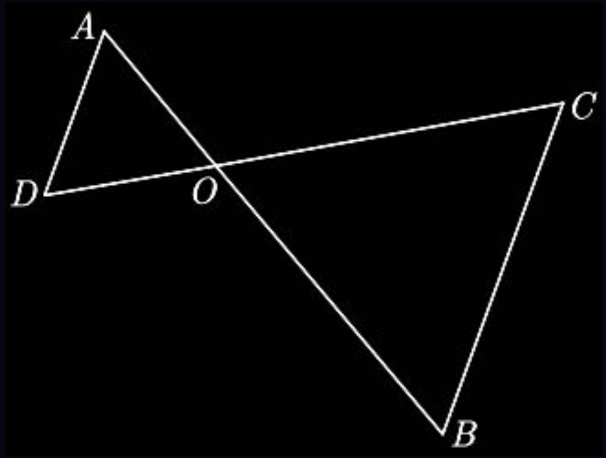
$$\Delta\, GEQ \sim GFS$$

$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

Q. In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$.

[Board 2011]



Solution

We have $$OA \times OB = OC \times OD$$
$$\frac{OA}{OD} = \frac{OC}{OB}$$

Due to the vertically opposite angles,

$$\angle AOD = \angle COB$$

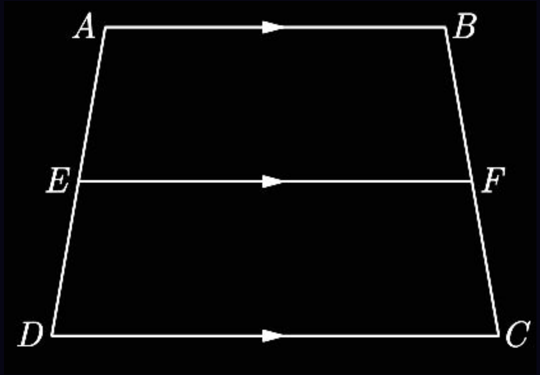
Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

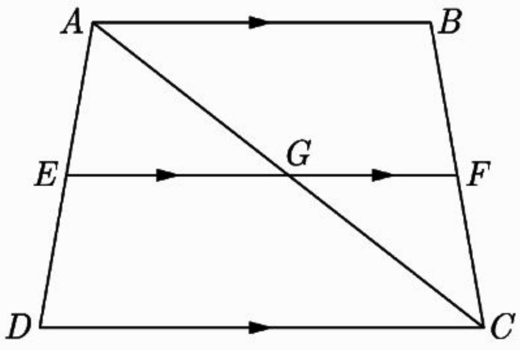
Q. In the given figure, if ABCD is a trapezium in which AB || CD || EF, then prove that AE/ED = BF/FC.

[Board 2012]



Solution

We draw, AC intersecting EF at G as shown below.



In $\Delta\, CAB$, $GF \,||\, AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \quad \text{...(1)}$$

In $\Delta\, ADC$, $EG \,||\, DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \quad \text{...(2)}$$

From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}.$$

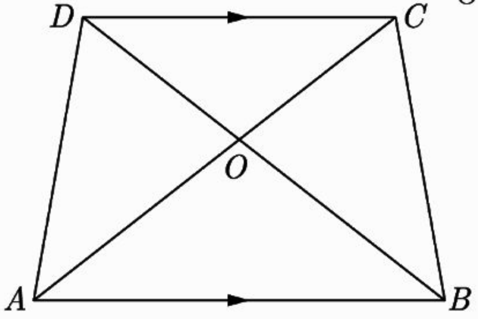
Hence Proved

Q. ABCD is a trapezium in which AB || CD and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO.

[Board 2013]

Solution

As per given condition we have drawn the figure below.



In $\Delta\, AOB$ and $\Delta\, COD$, $AB \,||\, CD$,

Thus due to alternate angles

$$\angle\, OAB = \angle\, DCO$$

and $$\angle\, OBA = \angle\, ODC$$

By AA similarity we have

$$\Delta\, AOB \sim \Delta\, COD$$

For corresponding sides of similar triangles we have

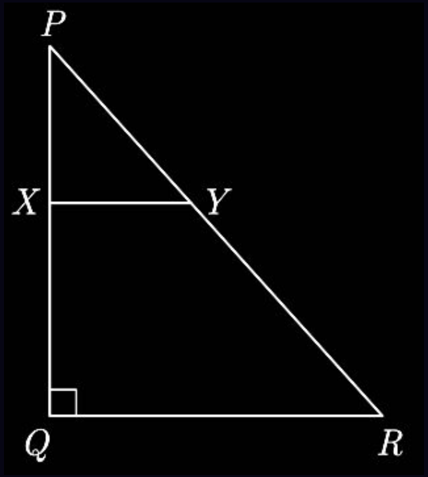
$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}.$$

Hence Proved

Q. In the given figure, PQR is a triangle right angled at Q and XY || QR. If PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the length of PR and QR.

[Board 2017]



Solution

Since $XY \parallel OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

In right ΔPQR we have

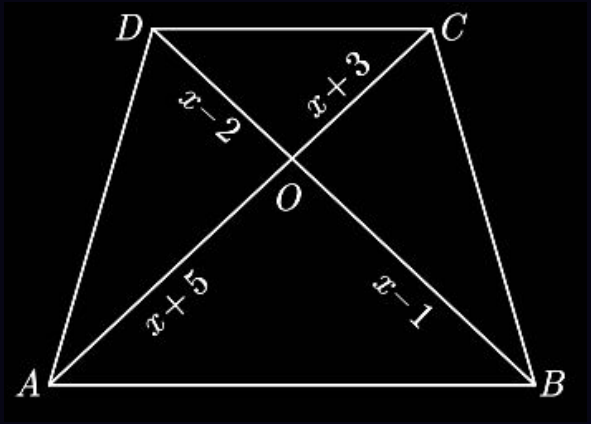
$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2 = 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

Q. In the given figure, if AB || DC, find the value of x.

[Board 2013]



Solution

Here ΔAOB is similar

to ΔCOD

because

$\angle CDO = \angle OBA$

$\angle DCO = \angle OAB$

Alternate interior angle an equal in || lines

$\angle DOC = \angle AOB$ (opposite angles are equal)

So its similar by (AAA)

(Angle Angle Angle) criteria.

So $\Delta AOB \sim \Delta COD$

$$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$$

from similatrily properly

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

Solution

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$(x+5)(x-2) = (x-1)(x+3)$$

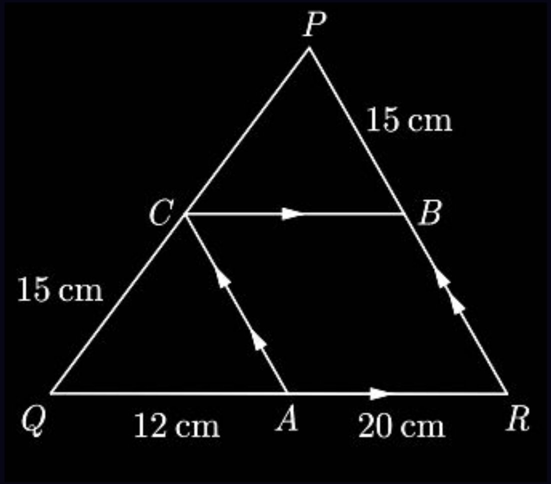
$$x^2 - 2x + 5x - 10 = x^2 - x + 3x - 3$$

$$3x - 10 = 2x - 3$$

$$\boxed{x = 7}$$

Q. In the given figure, CB || QR and CA || PR. If AQ = 12 cm, AR = 20 cm, PB = CQ = 15 cm, calculate PC and BR.

[Board 2011]



Solution

In ΔPQR, $CA \parallel PR$

By BPT similarity we have

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $CB \parallel QR$

Thus $\frac{PC}{CQ} = \frac{PR}{BR}$

$$\frac{25}{15} = \frac{15}{BR}$$

$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

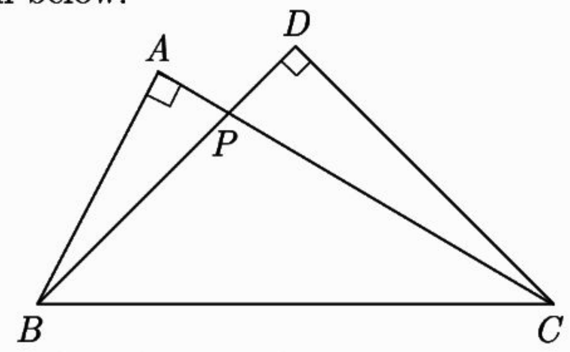
Q. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC . If AC and BD intersect at P, prove that AP ❌PC = BP ❌ DP.

[Board 2019]

Solution

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90^\circ$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

By AA similarity we have

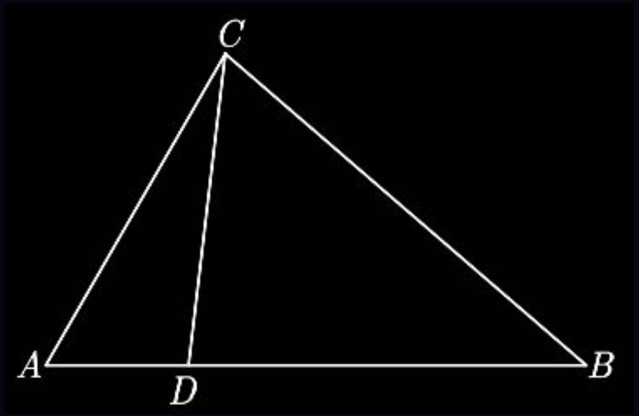
$$\Delta BAP \sim \Delta CDP$$

Therefore $$\frac{BP}{PC} = \frac{AP}{PD}$$

$$AP \times PC = BP \times PD$$ Hence Proved

Q. In the given figure, if ∠ACB = ∠CDA, AC = 6 cm and AD = 3 cm, then find the length of AB .

[Board 2020]



Solution

In ΔABC and ΔACD we have

$$\angle ACB = \angle CDA \quad \text{[given]}$$

$$\angle CAB = \angle CAD \quad \text{[common]}$$

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $$\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

Now $$\frac{AB}{AC} = \frac{AC}{AD}$$

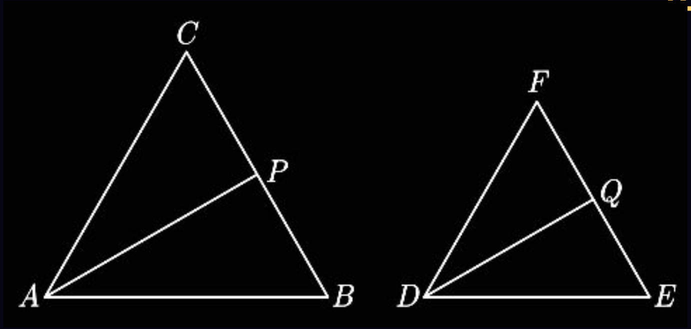
$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

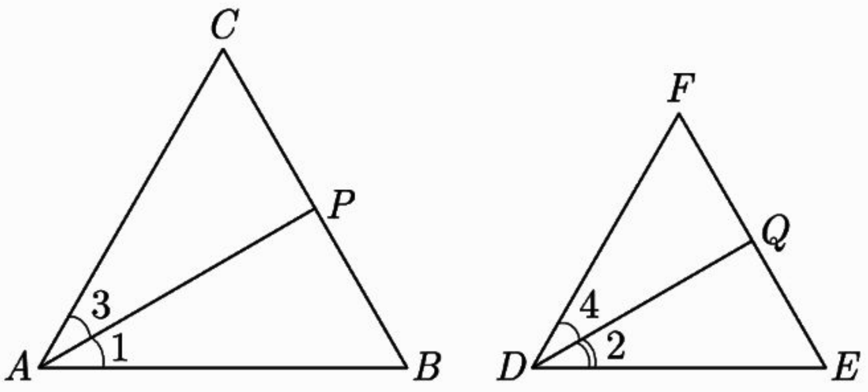
Q. In given figure $\triangle ABC \sim \triangle DEF$.
AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.
Prove that :
(1) AP/DQ = AB/DE
(2) $\triangle CAP \sim \triangle FDQ$.

[Board 2016]



Solution

As per given condition we have redrawn the figure below.



(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $$\angle B = \angle E \quad \text{(Corresponding angles)}$$

$$\frac{AP}{DQ} = \frac{AB}{DE}$$

Hence Proved

Solution

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $$\angle C = \angle F$$

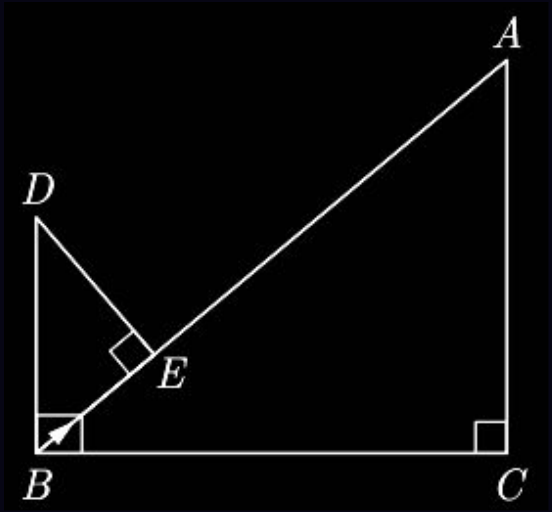
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

$$\Delta CAP \sim \Delta FDQ$$

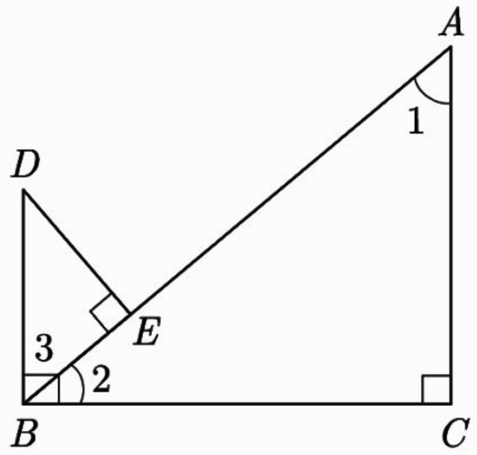
Q. In the given figure, DB ⊥ BC, DE ⊥ AB and AC ⊥ BC. Prove that BE/DE = AC/BC .



[Board 2011]

Solution

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In ΔABC, $\angle C = 90^\circ$, thus

$$\angle 1 + \angle 2 = 90^\circ$$

But we have been given,

$$\angle 2 + \angle 3 = 90^\circ$$

Solution

But we have been given,

$$\angle 2 + \angle 3 = 90^{o}$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

$$\angle 1 = \angle 3$$

and $\angle ACB = \angle DEB = 90^{o}$

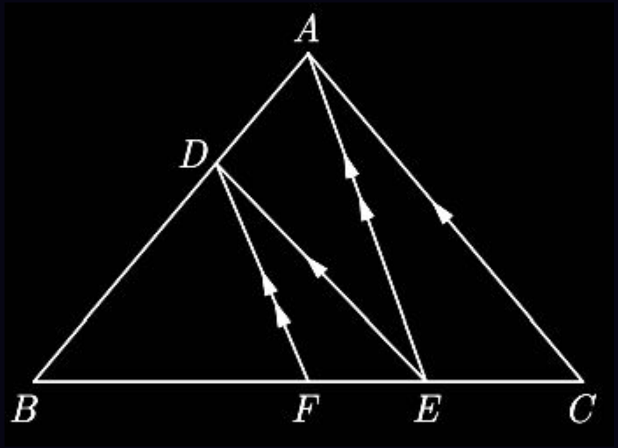
Thus by AA similarity we have

$$\Delta ABC \sim \Delta BDE$$

Thus $\dfrac{AC}{BC} = \dfrac{BE}{DE}$. Hence Proved

Q. In the given figure, DE || AC and DF || AE. Prove that BE/FE = BE/EC.

[Board 2020]



Solution

In $\Delta\, ABC$, $\quad DE \parallel AC,$ $\quad$ (Given)

By BPT $\quad \dfrac{BD}{DA} = \dfrac{BE}{EC}$ $\quad$ ...(1)

In $\Delta\, ABE$, $\quad DF \parallel AE,$ $\quad$ (Given)

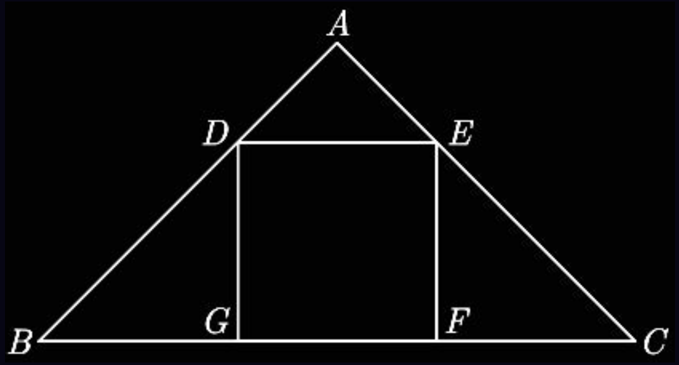
By BPT $\quad \dfrac{BD}{DA} = \dfrac{BF}{FE}$ $\quad$ ...(2)

From (1) and (2), we have

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

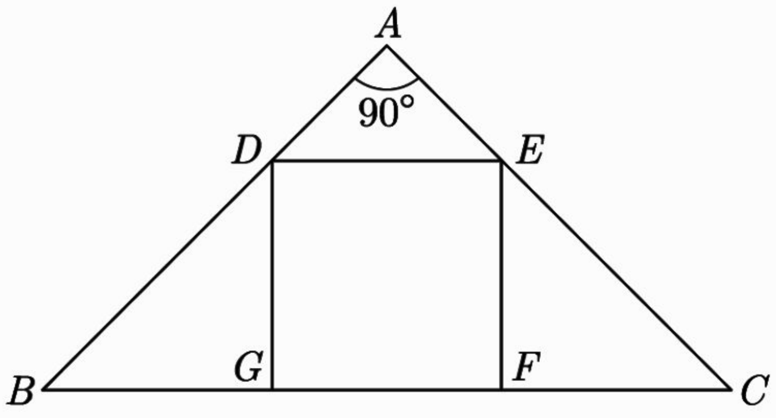
Q. In the given figure, DEFG is a square and $\angle BAC = 90°$. Show that $FG^2 = BG$ ❌ FC.

[SQP 2020]



Solution

We have redrawn the given figure as shown below.



In ΔADE and ΔGBD, we have

$$\angle DAE = \angle BGD \qquad \text{[each } 90^\circ\text{]}$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta GBD$$

Solution

Now, in ΔADE and ΔFEC,

$$\angle EAD = \angle CFE \qquad \text{[each } 90^\circ\text{]}$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta FEC$$

Since $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$ we have

$$\Delta GBD \sim \Delta FEC$$

Thus $$\frac{GB}{FE} = \frac{GD}{FC}$$

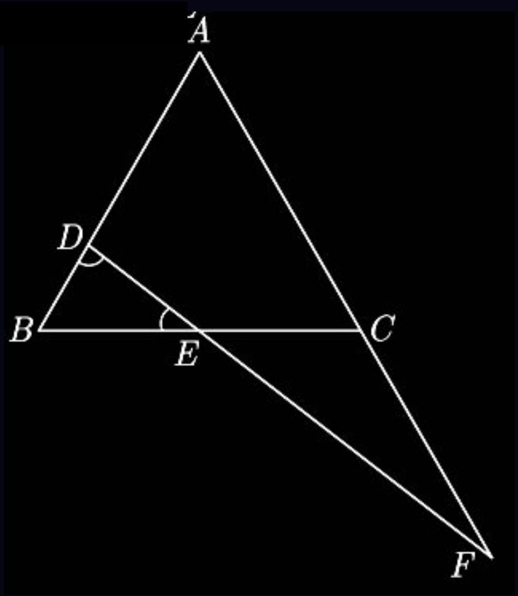
Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $$FG^2 = BG \times FC$$ Hence Proved

Q. In the figure, ∠BED = ∠BDE and E is the mid-point of BC. Prove that
AF/CF = AD/BE.

[Board 2019]



Solution

We have $\angle BED = \angle BDE$

Since E is mid-point of BC,

$$BE = BD = EC \qquad ...(1)$$

In ΔBCG, $DE \parallel FG$

From (1) we have

$$\frac{BD}{DG} = \frac{BE}{EC} = 1$$

$$BD = DG = EC = BE$$

In ΔADF, $CG \parallel FD$

By BPT $$\frac{AG}{GD} = \frac{AC}{CF}$$

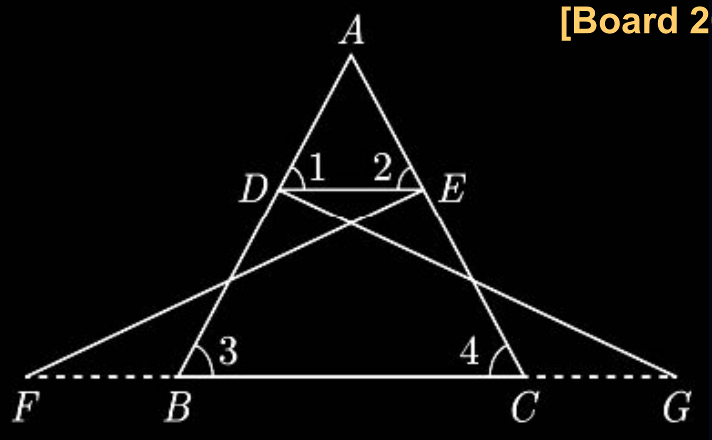
$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

, $$\frac{AD}{GD} = \frac{AF}{CF}$$

Thus $$\frac{AF}{CF} = \frac{AD}{BE}$$

Q. In the following figure,
▲ FEC ≅ ▲ GBD & ∠1 = ∠2. Prove that
▲ ADE ≅ ABC.

[Board 2011]



Solution

Since $\Delta FEC \cong \Delta GBD$

$$EC = BD \quad ...(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \quad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \parallel BC, \quad \text{(Converse of BPT)}$$

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 = \angle 4$$

Thus in ΔADE and ΔABC,

$$\angle A = \angle A$$

$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

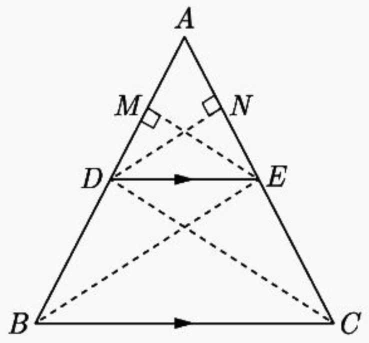
Thus by AAA criterion of similarity,

$$\Delta ADE \sim \Delta ABC$$ Hence proved

Q. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Solution

A triangle ABC is given in which $DE || BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD.



In $\Delta\, ADE$,

$$\text{area}\ (\Delta\, ADE) = \frac{1}{2} \times AE \times DN \qquad ...(1)$$

In $\Delta\, DEC$,

$$\text{area}\ (\Delta\, DCE) = \frac{1}{2} \times CE \times DN \qquad ...(2)$$

Dividing equation (1) by (2) we have,

$$\frac{\text{area}(\Delta\, ADE)}{\text{area}(\Delta\, DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or, $$\frac{\text{area}(\Delta\, ADE)}{\text{area}(\Delta\, DEC)} = \frac{AE}{CE} \qquad ...(3)$$

Solution

Now in ΔADE,

$$\text{area}(\Delta ADE) = \frac{1}{2} \times AD \times EM \qquad ...(4)$$

and in ΔDEB,

$$\text{area}(\Delta DEB) = \frac{1}{2} \times EM \times BD \qquad ...(5)$$

Dividing eqn. (4) by eqn. (5),

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or, $$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AD}{BD} \qquad ...(6)$$

Since ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$$\text{area}(\Delta DEB) = \text{area}(\Delta DEC)$$

From equation (3) we have

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AE}{CE} \qquad ...(7)$$

From equations (6) and (7) we get

$$\frac{AE}{CE} = \frac{AD}{BD}.$$

Hence proved.

Q. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that

🔺 ABC ~ 🔺 PQR.

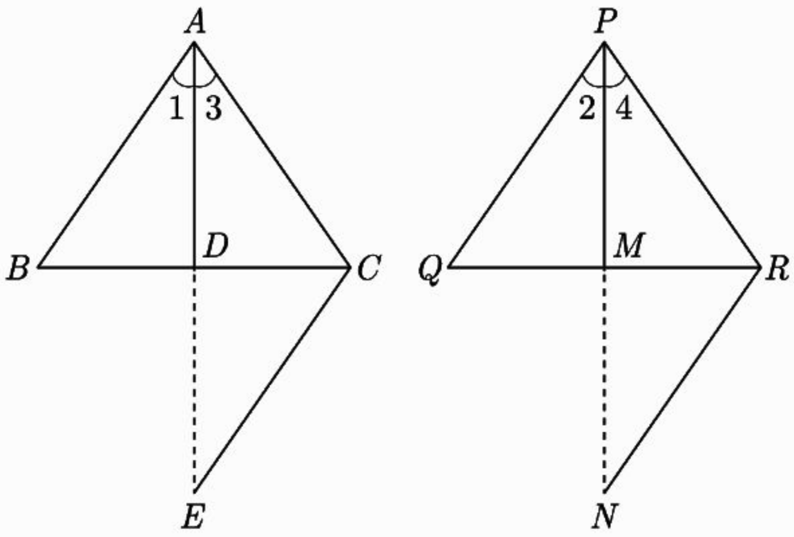
[Board 2012]

Solution

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. We join CE and RN. As per given condition we have drawn the figure below.



Solution

In $\Delta\, ABD$ and $\Delta\, EDC$,

$$AD = DE \qquad \text{(By construction)}$$

$$\angle ADB = \angle EDC \qquad \text{(VOA)}$$

$$BD = DC \qquad (AD \text{ is a median})$$

By SAS congruency

$$\Delta\, ABD \cong \Delta\, EDC$$

$$AB = CE \qquad \text{(By CPCT)}$$

Similarly, $PQ = RN$ and $\angle A = \angle 2$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \qquad \text{(Given)}$$

,

$$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta\, AEC \sim \Delta\, PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

By SAS similarity, we have

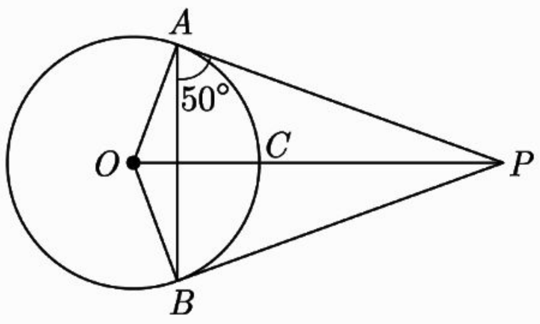
$$\Delta\, ABC \sim \Delta\, PQR \qquad \text{Hence Proved}$$

Geometry-Circles

Q. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^\circ$, then find $\angle AOB$.

[Board 2016]

Solution



Since $PA \perp OA$, $\angle OAP = 90^\circ$

$$\angle OAB = \angle OAP - \angle BAP$$
$$= 90^\circ - 50^\circ = 40^\circ$$

Since OA and OB are radii, we have

$$\angle OAB = \angle OBA = 40^\circ$$

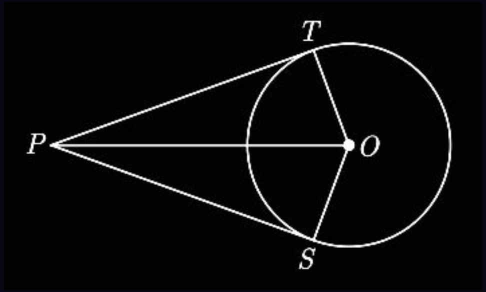
Now

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$
$$\angle AOB + 40^\circ + 40^\circ = 180^\circ$$
$$\angle AOB = 180^\circ - 80^\circ = 100^\circ$$

Hence $\angle AOB = 100^\circ$

Q. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that ∠SPT = 120°, Prove that OP = 2PS .

[Board 2016]



Solution

We have $\angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

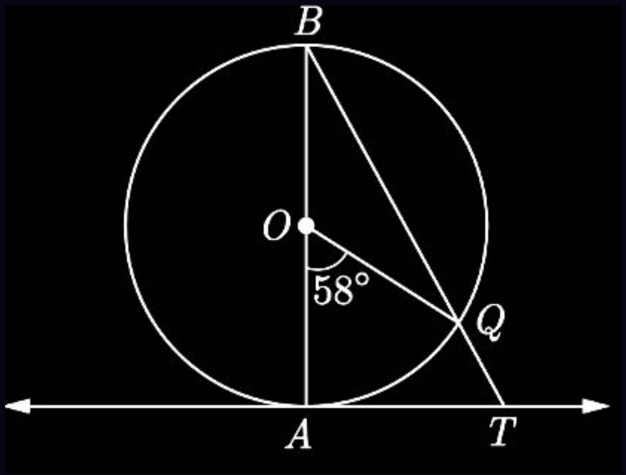
$$\frac{1}{2} = \frac{PS}{OP}$$

$$OP = 2PS$$

Hence proved.

Q. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.

[Board 2011]



Solution

We have $\angle AOQ = 58^\circ$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58^\circ = 29^\circ$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$$\angle ABQ = \angle ABT$$

Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

Thus $\angle ATQ = \angle ATB = 61^\circ$

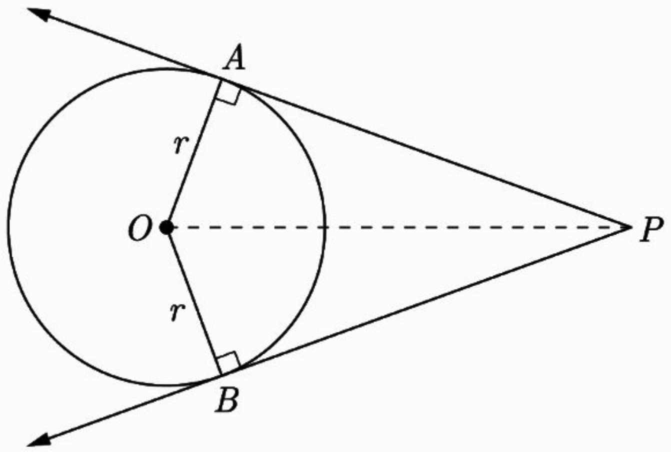
Q. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

[Board 2018]

Solution

Consider a circle of radius r and centre at O as shown in figure below. Here we have drawn two tangent from P at A and B. We have to prove that

$$AP = PB$$



We join OA, OB and OP. In ΔPAO and ΔPBO, OP is common and $OA = OB$ radius of same circle.

Since radius is always perpendicular to tangent, at point of contact,

$$\angle OAP = \angle OBP = 90^\circ$$

Solution

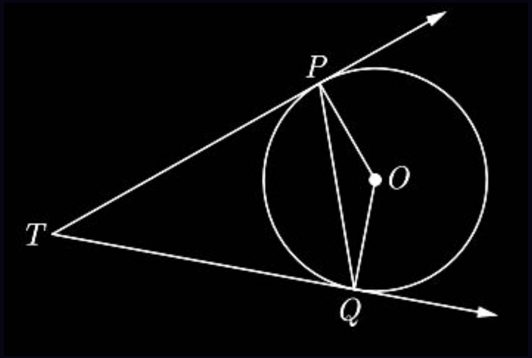
Thus $\Delta PAO \cong \Delta PBO.$

and hence, $AP = BP$

Thus length of 2 tangents drawn from an external point to a circle are equal.

Q. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find ∠PTQ.

[Board 2016]



Solution

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

Thus $\angle POQ = 60^\circ$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

$$\angle POQ + \angle PTQ = 180^\circ$$

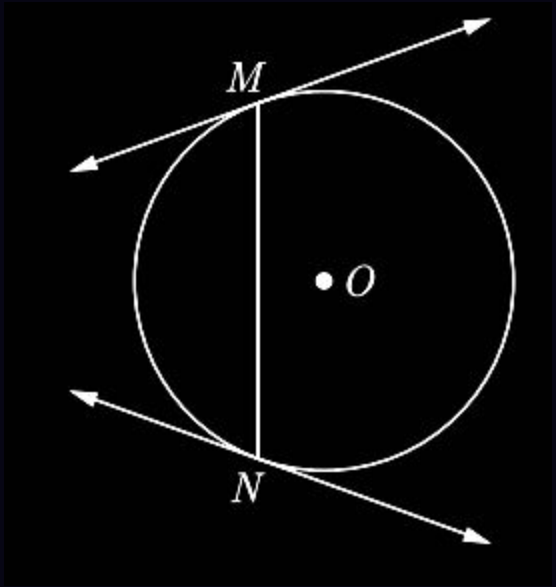
$$\angle PTQ = 180^\circ - \angle POQ$$

$$= 180^\circ - 60^\circ = 120^\circ$$

Thus $\angle PTQ = 120^\circ$

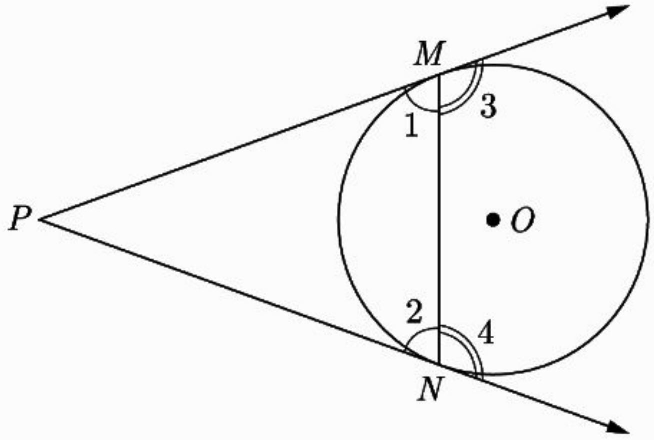
Q. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.

[Board 2015]



Solution

We redraw the given figure by joining M and N to P as shown below.



Since length of tangents from an external point to a circle are equal,

$$PM = PN$$

Since angles opposite to equal sides are equal,

$$\angle 1 = \angle 2$$

Now using property of linear pair we have

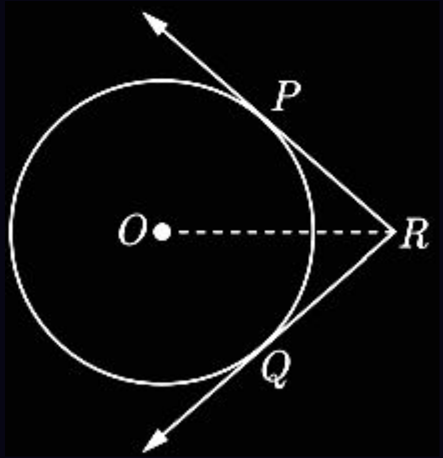
$$180^{\circ} - \angle 1 = 180^{\circ} - \angle 2$$

$$\angle 3 = \angle 4$$

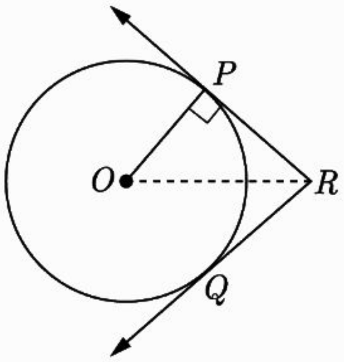
Hence Proved

Q. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that OR = PR + RQ.

[Board 2015]



Solution



$$\angle PRO = \tfrac{1}{2}\angle PRQ$$
$$= \frac{120^\circ}{2} = 60^\circ$$

Here ΔOPR is right angle triangle, thus

$$\angle POR = 90^\circ - \angle PRO = 90^\circ - 60^\circ = 30^\circ$$

Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$

$$OR = 2PR = PR + PR$$

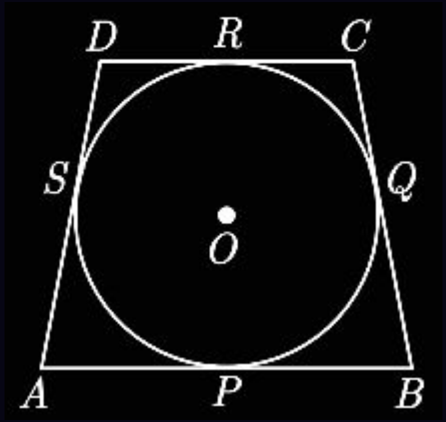
Since $PR = QR$,

$$OR = PR + QR$$ Hence Proved

Q. In Figure a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD, and DA touch the circle at the points P, Q, R and S respectively. Prove that

AB + CD = BC + DA.

[Board 2016]



Solution

Since length of tangents from an external point to a circle are equal,

At A,	$AP = AS$	(1)
At B	$BP = BQ$	(2)
At C	$CR = CQ$	(3)
At D	$DR = DS$	(4)

Adding eqn. (1), (2), (3), (4)

$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

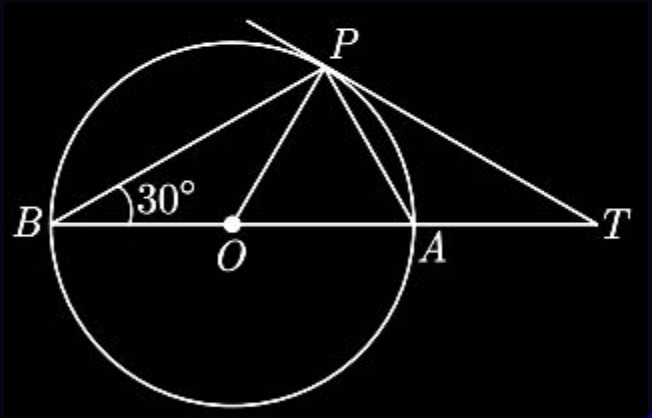
$$AP + BP + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Hence Proved

Q. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T.
If $\angle PBO = 30°$, what is the measure of $\angle PTA$?

[Board 2012]



Solution

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^0$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^0$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^\circ + 30^\circ = 60^\circ$$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^0$$

Now in right angle ΔOPT,

$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

Substituting $\angle OPT = 90^0$ and $\angle POT = 60^\circ$ we have

$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$

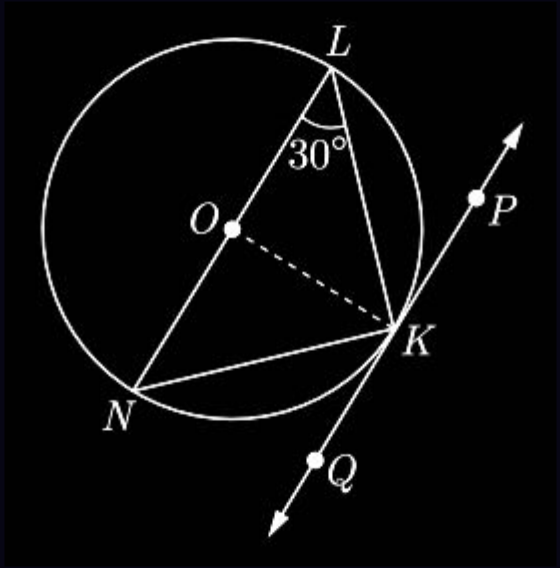
$$= 180^\circ - 150^\circ$$

$$= 30^\circ$$

Thus $\angle PTA = \angle PTO = 30^\circ$

Q. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and ∠KLN = 30°, find ∠PKL.

[Board 2017]



Solution

Since OK and OL are radius of circle, thus

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30^{\circ}$$

Tangent is perpendicular to the end point of radius,

$$\angle OKP = 90^{\circ} \qquad \text{(Tangent)}$$

Now $$\angle PKL = \angle OKP - \angle OKL$$

$$= 90^{\circ} - 30^{\circ} = 60^{\circ}$$

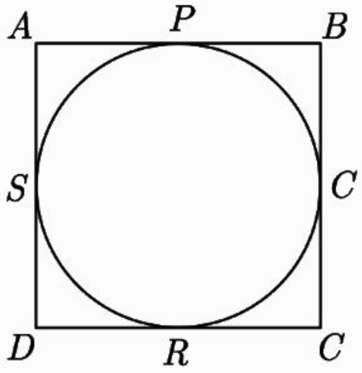
Q. Prove that the rectangle circumscribing a circle is a square.

[Board 2020]

Solution

We have a rectangle $ABCD$ circumscribe a circle which touches the circle at P, Q, R, S. We have to prove that $ABCD$ is a square.

As per given information we have drawn the figure below.



Since tangent drawn from an external point to a circle are equals,

$$AP = AS$$

$$PB = BQ$$

$$DR = DS$$

$$RC = QC$$

Solution

Adding all above equation we have

$$AP + PB + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Since $ABCD$ is rectangle, $AB = CD$ and $AD = BC$,

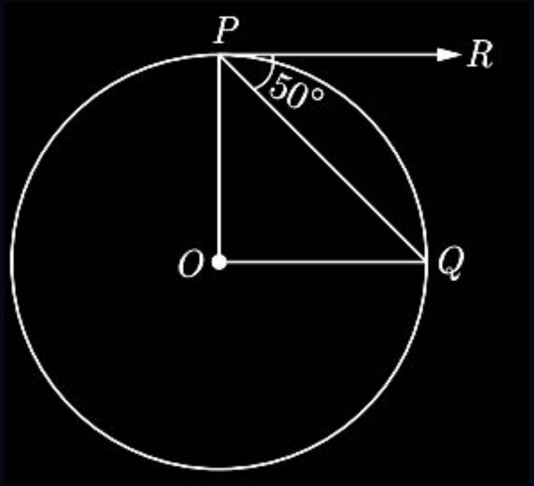
Thus $$2AB = 2BC$$

$$AB = BC$$

So, $ABCD$ is a square.

Q. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find ∠POQ.

[Board 2012]



Solution

We have $\angle RPQ = 50^\circ$

Since $\angle OPQ + \angle QPR$ is right angle triangle,

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Since, $OP = OQ$ because of radii of circle, we have

$$\angle OPQ = \angle OQR = 40^\circ$$

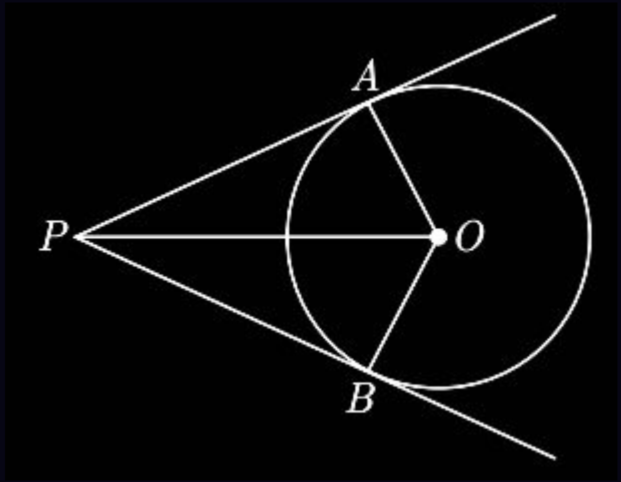
In ΔPOQ we have

$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$

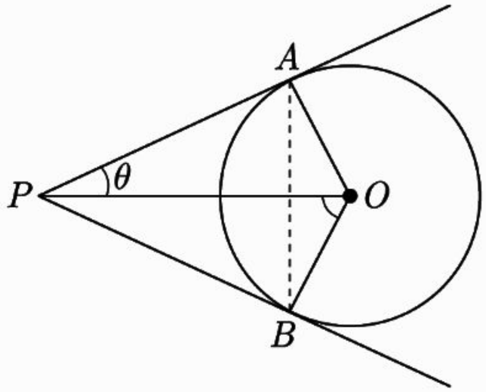
$$= 180^\circ - (40^\circ + 40^\circ) = 100^\circ$$

Q. In the given figure, OP is equal to the diameter of a circle with centre O and PA and PB are tangents. Prove that ABP is an equilateral triangle.

[Board 2014]



Solution



Since OA is radius and PA is tangent at A, $OA \perp AP$. Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$

$$\frac{OA}{OP} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

Hence, $\angle APB = 2 \times 30^\circ = 60^\circ$

Solution

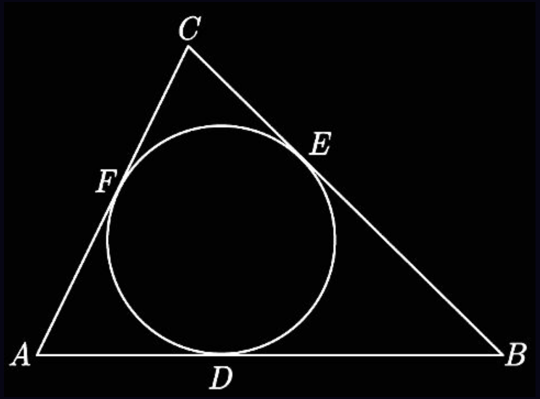
Now, in $\Delta\,APB$,

$$\begin{aligned} AP &= AB \\ \angle PAB &= \angle PBA \\ &= \frac{180^\circ - 60^\circ}{2} = 60^\circ \end{aligned}$$

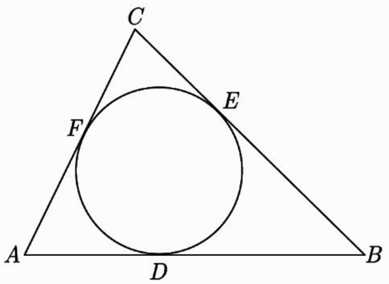
Thus $\Delta\,APB$ is an equilateral triangle.

Q. A circle is inscribed in a △ ABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

[Board 2013]



Solution



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$12 = 10 - x + 8 - x$

$2x = 18 - 12 = 6$

or $x = 3$

Now $AD = 3$ cm,

$BE = 10 - 3 = 7$ cm

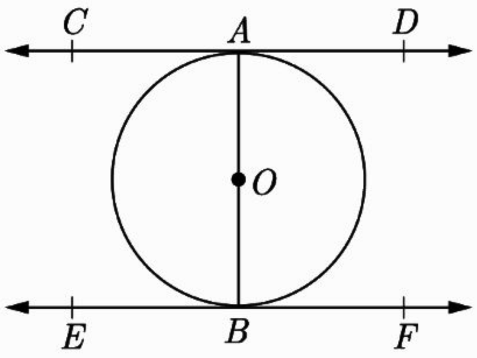
and $CF = 8 - 3 = 5$

Q. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

[Board 2017]

Solution

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^{\circ}$ and $\angle ABF = 90^{\circ}$

Hence $\angle CAB = \angle ABF$

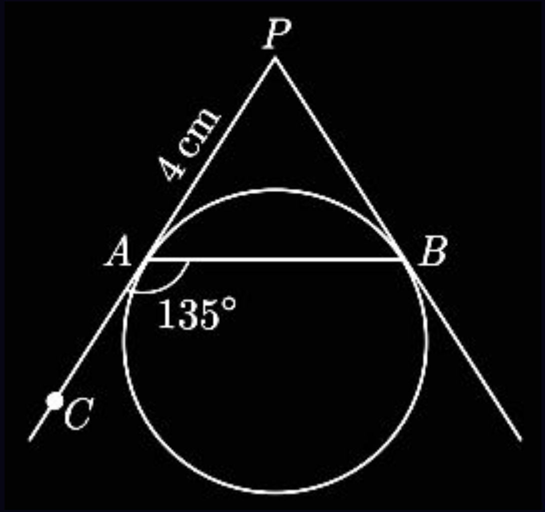
and $\angle ABE = \angle BAD$

Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD || EF$ Hence Proved

Q. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 4 cm and $\angle BAC = 135°$. Find the length of chord AB .

[Board 2017]



Solution

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

In triangle ΔAPB we have

$$\begin{aligned}\angle APB &= 180^\circ - \angle ABP - \angle BAP \\ &= 180^\circ - 45^\circ - 45^\circ \\ &= 90^\circ\end{aligned}$$

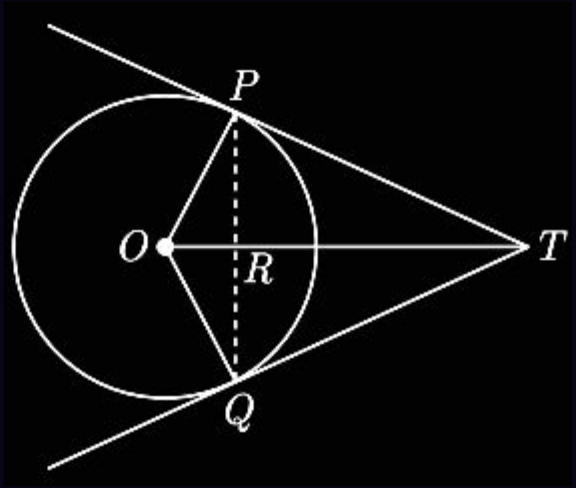
Thus ΔAPB is a isosceles right angled triangle

Now $$\begin{aligned}AB^2 &= AP^2 + BP^2 = 2AP^2 \\ &= 2 \times 4^2 = 32\end{aligned}$$

Hence $$AB = \sqrt{32} = 4\sqrt{2} \text{ cm}$$

Q. In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T . Find the length of TP.

[Board 2017]



Solution

Since length of tangents from an external point to a circle are equal,

$$PT = QT$$

Thus ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$.

Thus $OT \perp PQ$ and OT also bisects PQ.

Thus $$PR = PQ = \frac{8}{2} = 4 \text{ cm}$$

Since $\Delta\, OPR$ is right angled isosceles triangle,

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$

$$= 3 \text{ cm}$$

Now, Let $TP = x$ and $TR = y$ then we have

$$x^2 = y^2 + 16 \qquad (1)$$

Solution

Also in $\Delta\, OPT$,

$$x^2 + (5)^2 = (y+3)^2 \qquad (2)$$

Solving (1) and (2) we get

$$y = \frac{16}{3} \text{ and } x = \frac{20}{3}$$

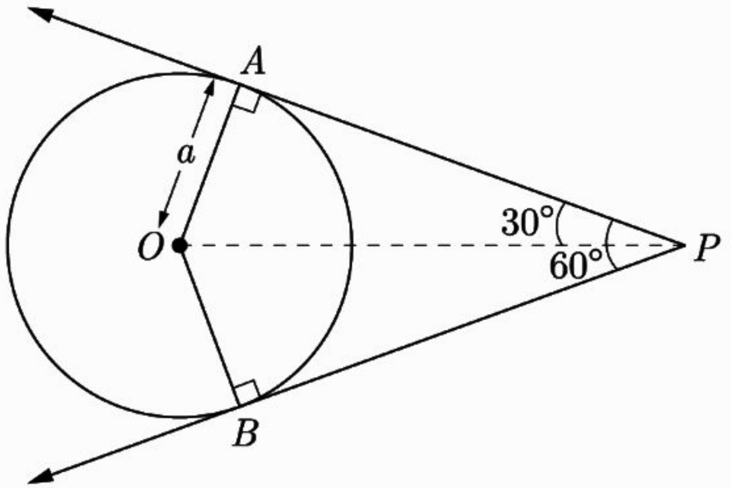
Hence, $TP = \dfrac{20}{3}$

Q. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

[Board 2020]

Solution

As per the given question we draw the figure as below.



Tangents are always equally inclined to line joining the external point P to centre O.

$$\angle APO = \angle BPO = \frac{60^\circ}{2} = 30^\circ$$

Also radius is also perpendicular to tangent at point of contact.

Solution

In right $\Delta\, OAP$ we have,

$$\angle APO = 30^{o}$$

Now,

$$\sin 30^{o} = \frac{OA}{OP}$$

Here OA is radius whose length is a, thus

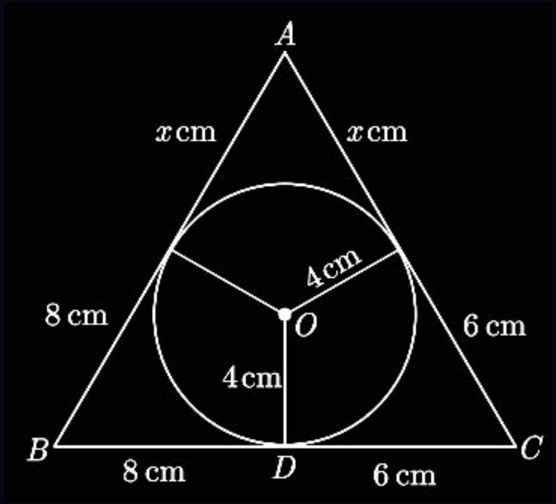
$$\frac{1}{2} = \frac{a}{OP}$$

or

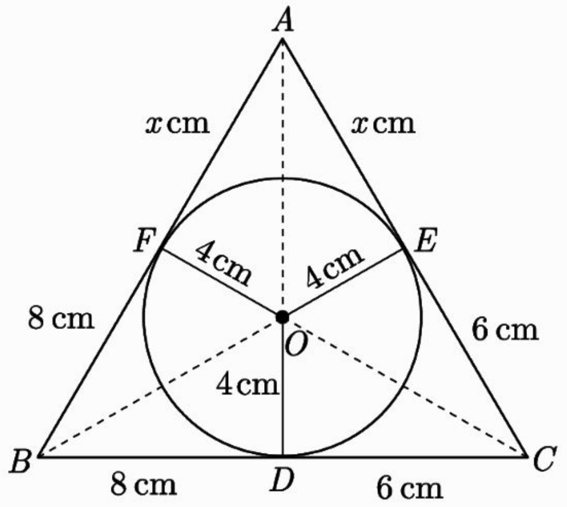
$$OP = 2a$$

Q. In the figure, the 🔺 ABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find AB and AC .

[Board 2014]



Solution



Since length of tangents from an external point to a circle are equal,

At A,	$AF = AE = x$	(2)
At B	$BF = BD = 8$ cm	(3)
At C	$CD = CE = 6$ cm	(4)
Now	$AB = x + 8$	

Solution

Now
$$AB = x + 8$$
$$AC = x + 6$$
$$BC = 8 + 6 = 14 \text{ cm}$$

Perimeter of circle

$$p = AB + BC + CA$$
$$= x + 8 + 14 + x + 6$$
$$= 2(x + 14)$$

Semi-perimeter of circle

$$s = \frac{1}{2}p = x + 14$$

Area or triangle ΔABC

$$\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48x^2 + 672x} \quad (1)$$

Solution

Area or triangle ΔABC,

$$\Delta ABC = \frac{1}{2} rp$$

$$= \frac{1}{2} \times 4 \times 2(x + 14)$$

$$= 4(x + 14) \qquad (2)$$

From equation (1) and (2) we have

$$48x^2 + 672x = 16(x + 14)^2$$

$$48x(x + 14) = 16(x + 14)^2$$

$$3x = x + 14$$

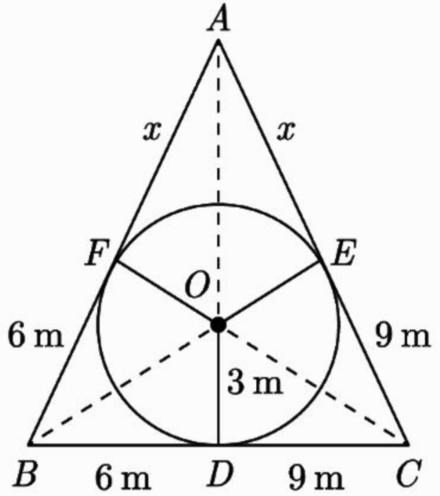
or, $$x = 7$$

Hence, AB = 15 cm & AC = 13 cm

Q. If BD = 6 m, DC = 9 m and ar(△ ABC) = $54\ m^2$, then find the length of sides AB and AC respectively.

Solution

Now we have redrawn the figure as shown below.



Since tangents drawn from external points are of equal length. So,

$$BF = BD = 6\,\text{m}$$

$$CE = CD = 9\,\text{m}$$

Let $\quad AF = AE = x\,\text{m}$

Solution

Now,

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta AOB) + \text{ar}(\Delta BOC) + \text{ar}(\Delta AOC)$$

$$54 = \frac{1}{2}AB \times OF + \frac{1}{2}BC \times OD + \frac{1}{2}AC \times OE$$

$$54 = \frac{1}{2}(6+x) \times 3 + \frac{1}{2}(6+9) \times 3 + \frac{1}{2}(9+x) \times 3$$

$$54 = \frac{3}{2}[6 + x + 15 + 9 + x]$$

$$36 = 2x + 30 \quad \rightarrow \quad x = 3$$

So, $AB = x + 6 = 9\,\text{m}$

and $AC = x + 9 = 12\,\text{m}$

Q. Find the perimeter of Δ ABC .

Solution

$$\begin{aligned} \text{Perimeter of } \Delta ABC \qquad s &= AB + BC + AC \\ &= 9 + 15 + 12 \\ &= 36 \text{ m} \end{aligned}$$

Trigonometry

Q. If cos X = $\frac{2}{3}$, then tan X=

Solution

By trigonometry identities, we know

$$1 + \tan^2 X = \sec^2 X$$

And $\sec X = \dfrac{1}{\cos X} = \dfrac{1}{\left(\frac{2}{3}\right)} = \dfrac{3}{2}$

Hence,

$$1 + \tan^2 X = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\tan^2 X = \left(\frac{9}{4}\right) - 1 = \frac{5}{4}$$

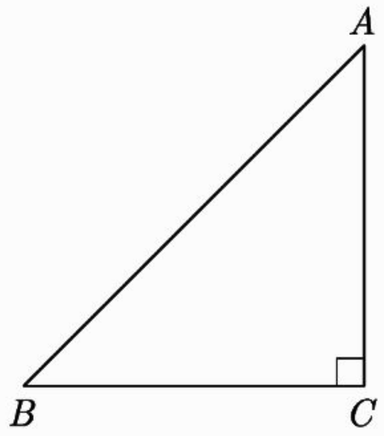
$$\tan X = \frac{\sqrt{5}}{2}$$

Q. If △ABC is right angled at C, then the value of cos(A + B) is

[SQP 2022]

Solution

We know that in ΔABC,



$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$A + B = 90^\circ$$

$$\cos(A+B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

Q. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, ($\theta \neq 90°$) then the value of $\tan\theta$ is

Solution

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$$

$$\tan\theta + 1 = \sqrt{2}$$

$$\tan\theta = \sqrt{2} - 1$$

Thus (a) is correct option.

Q. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2$ is equal to

Solution

Given, $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$$

$$= 4\tan\theta\sin\theta$$

$$= 4\sqrt{\tan^2\theta\sin^2\theta}$$

$$= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\sin^2\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{mn}$$

Thus (c) is correct option.

Q. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is is equal to

Solution

Given, $\sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

$$\sin\theta = \sin(90^\circ - \theta)$$

$$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Thus (c) is correct option.

Q. If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos \theta$, then $x^2 + y^2$ is equal to

Solution

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$$

Now, $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta$$

$$y = \sin\theta$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Thus (c) is correct option.

Q. If 🔺ABC is right angled at C , then the value of sec(A + B) is

Solution

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

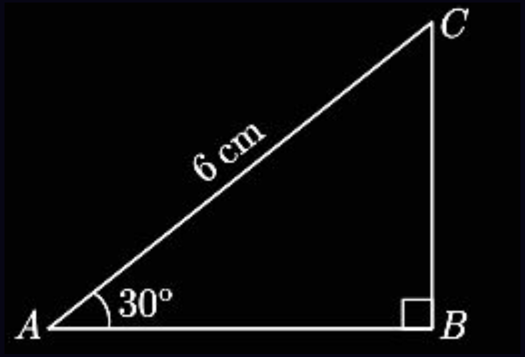
$$\angle A + \angle B = 180^\circ - \angle C$$

$$= 180^\circ - 90^\circ = 90^\circ$$

Now, $\sec(A + B) = \sec 90^\circ$ not defined

Thus (d) is correct option.

Q. In the adjoining figure, the length of BC



Solution

In ΔABC,

$$\sin 30^\circ = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3\,\text{cm}$$

Thus (d) is correct option.

Q. If $4 \tan \theta = 3$, then $\dfrac{(4\sin\theta - \cos\theta)}{(4\sin\theta + \cos\theta)}$

Solution

Given, $$4\tan\theta = 3$$

$$\tan\theta = \frac{3}{4} \quad \ldots\text{(i)}$$

$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

Q. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

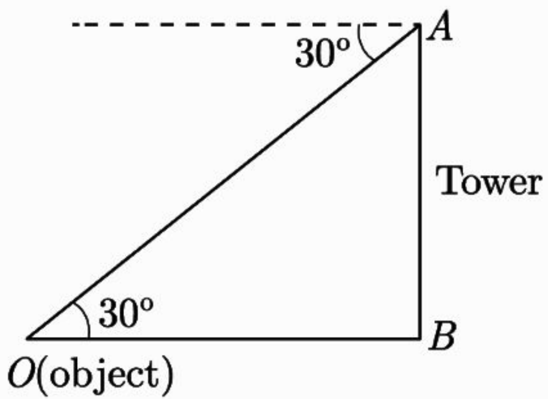
Solution

We have

$$\tan 30^\circ = \frac{AB}{OB}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$

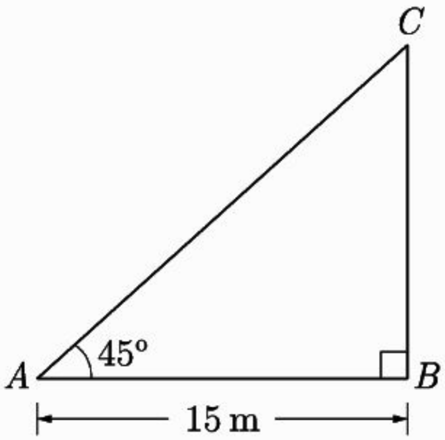


Thus (c) is correct option.

Q. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45c. The height of a tree is

Solution

Let BC be the tree of height h meter. Let AB be the shadow of tree.



In ΔABC, $\quad CB = 90^\circ$

$$\frac{BC}{BA} = \tan 45^\circ$$

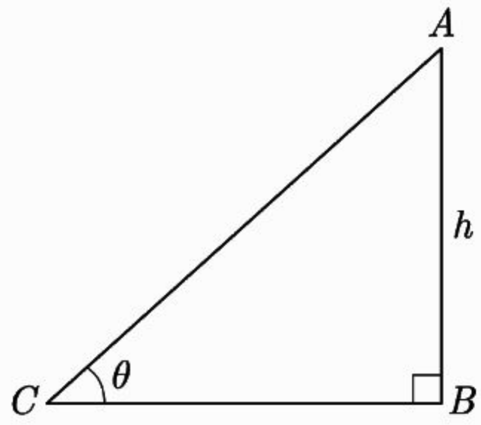
$$BC = AB = 15\,\text{m}$$

Thus (d) is correct option.

Q. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$ then the angle of elevation of the sun is

Solution

Let AB be the rod of length h, BC be its shadow of length $\sqrt{3}\,h$, θ be the angle of elevation of the sun.



In $\Delta\,ABC$, $$\frac{h}{\sqrt{3}\,h} = \tan\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Q. A 6 m high tree cast a 4 m long shadow. At the same time, a flagpole cast a shadow 50 m long. How long is the flag pole?

Solution

According to the concept used

Both triangle will be similar

So, Ratio of their sides will be equal

$6 : 4 = ? : 50$

$\Rightarrow 6/4 = ?/50$

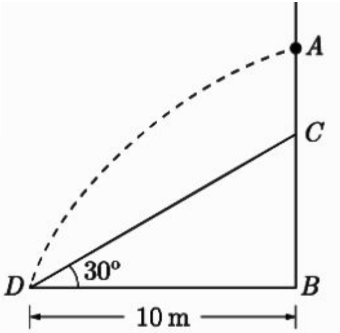
$\Rightarrow ? = (6/4) \times 50$

$\Rightarrow ? = 75$

∴ Height of the long pole is 75m.

Q. A tree is broken by the wind. The top struck the ground at an angle of 30° and at distance of 10 m from its root. The whole height of the tree is ($\sqrt{3}$ = 1.732)

Solution



Now $AC = CD$

$\angle CDB = 30^\circ$

$BD = 10\text{ m}$

In ΔCDB, $\tan 30^\circ = \frac{CB}{DB} = \frac{CB}{10}$

$$\frac{1}{\sqrt{3}} = \frac{CB}{10}$$

$$CB = \frac{10}{\sqrt{3}}$$

Also, $\cos 30^\circ = \frac{DB}{DC} = \frac{10}{DC}$

$$DC = \frac{20}{\sqrt{3}} = AC$$

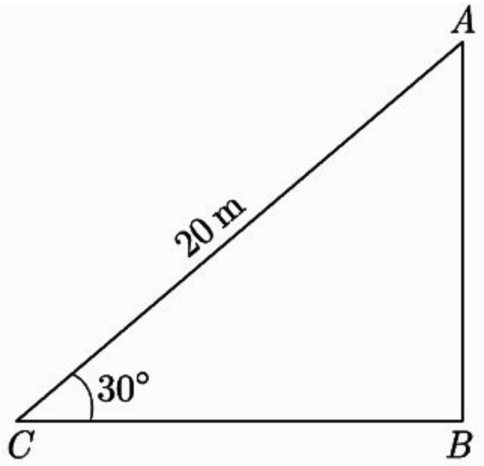
Height of tree,

$$AC + CB = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3}\text{ m}$$

Q. A circle artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the angle made by the rope with the ground level is 30°, is

Solution



In $\triangle ABC$, we have

$$\sin 30^\circ = \frac{AB}{AC}$$

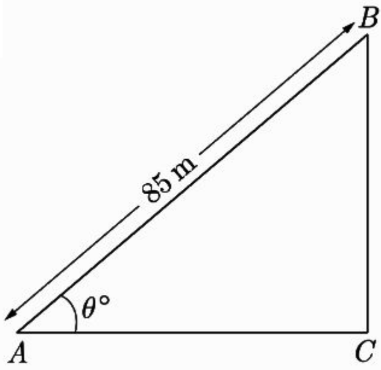
$$\frac{1}{2} = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

Q. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that tan θ = 15/8 , then the height of kite is

Solution



Length of the string of the kite,

$$AB = 85\text{ m}$$

and $$\tan\theta = \frac{15}{8}$$

$$\cot\theta = \frac{8}{15}$$

$$\operatorname{cosec}^2\theta - 1 = \frac{64}{225}$$

$$\operatorname{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$$

$$\operatorname{cosec}\theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$$

$$\sin\theta = \frac{15}{17}$$

In $\triangle ABC$, $$\sin\theta = \frac{BC}{AB}$$

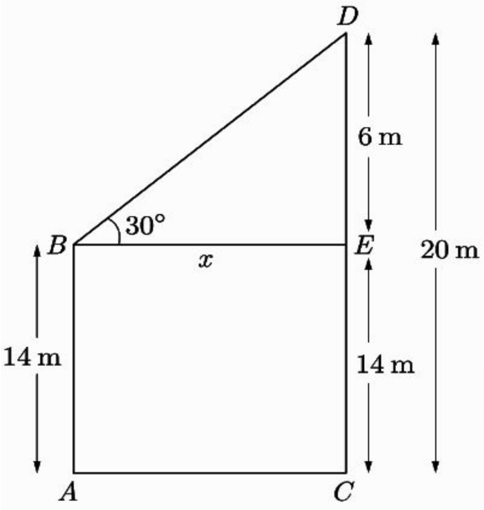
$$\frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75\text{ m}$$

Thus height of kite is 75 m.

Q. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

Solution

Height of big pole	$CD = 20\text{ m}$
Height of small pole	$AB = 14\text{ m}$



$$DE = CD - CE$$
$$= CD - AB \qquad [AB = CE]$$
$$= 20 - 14 = 6\text{ m}$$

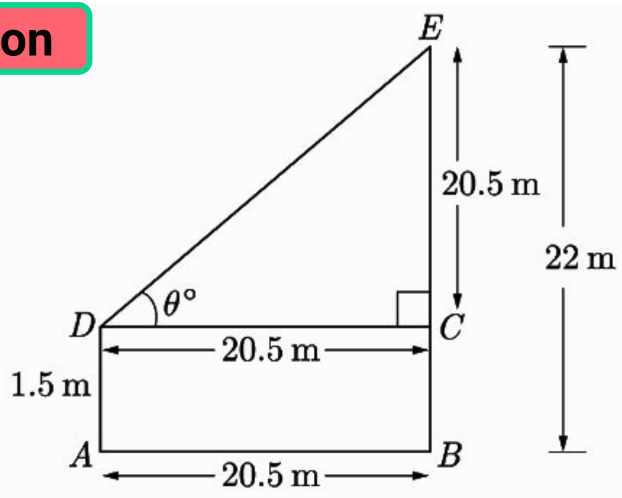
In $\triangle BDE$, $\sin 30^\circ = \dfrac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12\text{ m}$$

Thus length of wire is 12 m.

Q. An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of observer is

Solution



Then, $AB = 20.5\text{ m} = DC$

and $EC = BE - BC = BE - AD$

$= 22 - 1.5 = 20.5\text{ m}$ $[BC = AD]$

Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle DCE$,

$$\tan\theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$

$$\tan\theta = 1$$

$$\tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

Q. A lamp post $5\sqrt{3}$ m high casts a shadow 5 m long on the ground. The sun's elevation at this point is:

Solution

Let the angle of elevatoin be $'\theta'$.

In, ΔACB,

$$\Rightarrow \tan\theta = \frac{AB}{BC} = \frac{5\sqrt{3}}{5}$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \ Or\ \theta = 60^o$$

Hence, the answer is 60^o

Q. If $3 \cot \theta = 2$, find the value of

$$\frac{2 \sin \theta - 3 \cos \theta}{2 \sin \theta + 3 \cos \theta}$$

[Board 2011]

Solution

$$3\cot\theta = 2 \qquad \Rightarrow \cot\theta = \frac{2}{3}$$

$$\text{Required expression} = \frac{2\sin\theta - 3\cos\theta}{2\sin\theta + 3\cos\theta}$$

$$= \frac{\frac{2\sin\theta}{\sin\theta} - \frac{3\cos\theta}{\sin\theta}}{\frac{2\sin\theta}{\sin\theta} + \frac{3\cos\theta}{\sin\theta}} = \frac{2-3\cot\theta}{2+3\cot\theta}$$

$$= \frac{2-3\times\frac{2}{3}}{2+3\times\frac{2}{3}} = \frac{2-2}{2+2} = \frac{0}{4} = 0$$

Q. If $\tan\theta + \cot\theta = 5$, find the value of $\tan^2\theta + \cot^2\theta$

[Board 2012]

Solution

$\tan\theta + \cot\theta = 5$

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$

...[Squaring both sides

$\tan^2\theta + \cot^2\theta + 2 = 25$

$\tan^2\theta + \cot^2\theta = 23$

Q. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

[Board 2012]

Solution

Given: $cos\theta + sin\theta = \sqrt{2}\cos\theta$

$\sin\theta = \sqrt{2}\ \cos\theta - \cos\theta$

$\sin\theta = \cos\theta\left(\sqrt{2} - 1\right) \qquad \ldots(i)$

To Prove. $cos\theta - sin\theta = \sqrt{2}\ \sin\theta$

$\therefore\ \cos\theta = \sqrt{2}\ \sin\theta + \sin\theta$

$\Rightarrow \cos\theta = \sin\theta\left(\sqrt{2} + 1\right) \qquad \ldots(ii)$

Putting value of $\sin\theta$ from

(i) into (ii), we get

$cos\theta = cos\theta\left(\sqrt{2} - 1\right)\left(\sqrt{2} + 1\right)$

$\Rightarrow\ \cos\theta = \cos\theta$, which is true

Q. If $5 \sin \theta = 4$, rove that $\frac{1}{\cos \theta} + \frac{1}{\cot \theta} = 3.$

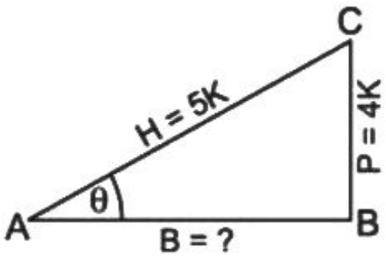
[Board 2013]

Solution

Given: $5 sin\,\theta = 4$

$\therefore\ \sin\theta = \frac{4}{5}\quad \therefore\ \frac{P}{H} = \frac{4}{5}$

$P = 4K,\ H = 5K$



In rt. ΔABC

$P^2 + B^2 = H^2$ ... [by Pythagoras' theorem

$(4K)^2 + B^2 = (5K)^2$

$B^2 = 25K^2 - 16K^2 = 9K^2$

$B = +3K$ ... [$\because$ Base (B) cannot be -ve

$sec\,\theta = \frac{1}{\cos\theta} = \frac{H}{B} = \frac{5K}{3K} = \frac{5}{3},$

$\tan\theta = \frac{P}{B} = \frac{4K}{3K} = \frac{4}{3}$

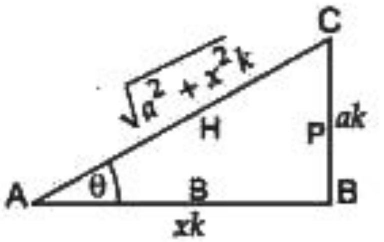
$LHS = \frac{1}{\cos\theta} + \frac{1}{\cot\theta} = \sec\theta + \tan\theta$

$= \frac{5}{3} + \frac{4}{3} = \frac{9}{3} = 3 = RHS$

Q. If $\tan\theta = \frac{a}{x}$, find the value of $\frac{x}{\sqrt{a^2 + x^2}}$.

[Board 2013]

Solution



$$\tan\theta = \frac{a}{x}$$

$$\frac{P}{B} = \frac{a}{x}$$

$$\therefore P = ak,\ \ B = xk$$

In rt. ΔABC,

$$H^2 = P^2 + B^2$$

$$= a^2k^2 + x^2k^2 = k^2\left(a^2 + x^2\right)$$

$$\therefore H = k\sqrt{a^2 + x^2}$$

$$\frac{x}{\sqrt{a^2+x^x}} = \frac{AB}{AC} = \frac{B}{H} = \cos\theta$$

Q. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

[Board 2013]

Solution

$$\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$$

$$\sqrt{3}\cot^2\theta + \sqrt{3} = 4\cot\theta$$

$$\sqrt{3}\left(\cot^2\theta + 1\right) = 4\cot\theta$$

$$\frac{\cot^2\theta+1}{\cot\theta} = \frac{4}{\sqrt{3}} \Rightarrow \frac{\cot^2\theta}{\cot\theta} + \frac{1}{\cot\theta} = \frac{4}{\sqrt{3}}$$

$$\cot\theta + \tan\theta = \frac{4}{\sqrt{3}}$$

Squaring both sides

$$\cot^2\theta + \tan^2\theta + 2\cot\theta\tan\theta = \frac{16}{3}$$

$$\cot^2\theta + \tan^2\theta + 2 = \frac{16}{3} \quad \ldots [\because \cot\theta.\tan\theta = 1]$$

$$\cot^2\theta + \tan^2\theta = \frac{16}{3} - 2 = \frac{16-6}{3} = \frac{10}{3}$$

Q. If $\sin\theta = \frac{12}{13}$, 0° < θ < 90°, find the value of: $\dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \dfrac{1}{\tan^2\theta}$

[Board 2015]

Solution

Given: $\sin\theta = \dfrac{12}{13} \quad \therefore \dfrac{P}{H} = \dfrac{12}{13}$

Let, $P = 12K, \quad H = 13K$

$P^2 + B^2 = H^2 \quad \ldots$ [Pythagora's theorem]

$(12K)^2 + B^2 = (13K)^2$

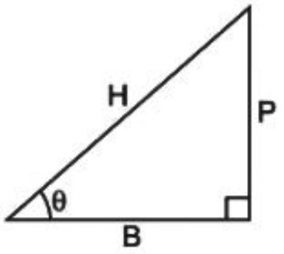
$144K^2 + B^2 = 169K^2$

$B^2 + 169K^2 - 144K^2 = 25K^2$

$B = 5K$

$\therefore \cos\theta = \dfrac{B}{H} = \dfrac{5k}{13K} = \dfrac{5}{13}$

$\tan\theta = \dfrac{P}{B} = \dfrac{12K}{5K} = \dfrac{12}{5}$



Solution

$$\text{Now, } \frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}$$

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144} = \frac{119}{120} \times \frac{25}{144} = \frac{595}{3,456}$$

Q. Prove that: $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.$

[Board 2015]

Solution

$$LHS = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left[2\left(1 - \sin^2\theta\right) - 1\right]} \quad \ldots \left[\because\ \cos^2\theta = 1 - \sin^2\theta\right.$$

$$= \frac{\tan\theta\left(1 - 2\sin^2\theta\right)}{\left(2 - 2\sin^2\theta - 1\right)} = \frac{\tan\theta\left(1 - 2\sin^2\theta\right)}{\left(1 - 2\sin^2\theta\right)}$$

$$= \tan\theta = RHS\ \left(\text{Hence proved}\right)$$

Q. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then $x =$

[SQP 2020]

Solution

Given, $x \ \tan 60^0 \cos 60^0 = \sin 60^o \cot 60^o$

$$x \ \tan 60^0 \cos 60^0 = \sin 60^0 \times \frac{1}{\tan 60^0}$$

Putting values

$$x \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$

$$x \times \frac{\sqrt{3}}{2} = \frac{1}{2}$$

$$x = \frac{1}{2} \times \frac{2}{\sqrt{3}}$$

$$x = \frac{1}{\sqrt{3}}$$

Q. The value of

$\cos 1^\circ . \cos 2^\circ . \cos 3^\circ . \cos 4^\circ . . . . \cos 90^\circ$ is

[SQP 2022]

Solution

cos1° cos2° cos3° ______ cos90°

$\therefore$ (cos1°) × (cos2°) × (cos3°) × ______ × (cos89°) × (cos90°)

cos90° = 0

$\therefore$ (cos1°) × (cos2°) × (cos3°) × ______ × (cos89°) × (cos90°) = 0

Q. If sin(A + B) = 1 and

$$\cos(A - B) = \frac{\sqrt{3}}{2},\ 0^o < A + B \leq 90^o$$ and

A > B, then find the measures of angles A and B.

[SQP 2022]

Solution

$\sin(A+B) = 1 = \sin 90,\ \text{so}\ A+B = 90\ \ldots\ldots (i)$

$\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30, \text{so}\ A-B = 30\ \ldots. (ii)$

From (i) & $(ii) \angle A = 60^o$

And $\angle B = 30^o$

Q. Find an acute angle θ when

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

[SQP 2022]

Solution

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing the numerator and denominator of LHS by $\cos\theta$, we get

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Which on simplification (or comparison) gives $\tan\theta = \sqrt{3}$

Or $\theta = 60^o$

Q. If sin A = cos B = $\frac{1}{2}$ Find the value of tan(A + B)

[CBSE 2012]

Solution

$$\sin A = \frac{1}{2}; \qquad \cos B = \frac{1}{2}$$

$$\therefore\ A = 30^\circ \qquad B = 60^\circ$$

$$\tan(A+B) = \tan(30^\circ + 60^\circ) = \tan 90^\circ$$

Not defined.

Q. Evaluate $10.\dfrac{1-\cot^2 45^0}{1+\sin^2 90^0}.$

[CBSE 2014]

Solution

$$10.\frac{1-\cot^{2}45^{0}}{1+\sin^{2}90^{0}}$$

$$=10.\frac{1-(1)^{2}}{1+(1)^{2}}$$

$$=10.\left(\frac{0}{2}\right)=0$$

Q. If $\sqrt{3}\sin\theta = \cos\theta$,

find the value of $\dfrac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}$.

[CBSE 2015]

Solution

Given: $\sqrt{3}\sin\theta = \cos\theta$

$$\frac{1}{\sqrt{3}} = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \tan 30^\circ \Rightarrow \theta = 30^\circ \quad \ldots(i)$$

Now, $\dfrac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2} = \dfrac{\cos\theta\,(3\cos\theta + 2)}{(3\cos\theta + 2)}$

$$= \cos\theta = \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \ldots[\text{From (i)}]$$

Q. If $\tan\alpha = \sqrt{3}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, $0 < \alpha, \beta < 90°$ then find the value of $\cot(\alpha + \beta)$.

[CBSE 2017]

Solution

$\tan\alpha = \sqrt{3} = \tan 60^\circ;\ \tan\beta = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\therefore\ \alpha = 60^\circ$ and $\beta = 30^\circ$

Hence, $cot\,(\alpha+\beta) = \cot\,(60^\circ + 30^\circ) = \cot 90^\circ = 0$

Q. If $\sin\theta - \cos\theta = 0$, then find the value of $\sin^4\theta + \cos^4\theta$.

[CBSE 2017]

Solution

$$\sin\theta - \cos\theta = 0 \quad \Rightarrow \sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1 \quad \Rightarrow \tan\theta = 1 \;\Rightarrow \theta = 45^\circ$$

Now, $sin^4\theta + \cos^4\theta$

$$= \sin^4 45^\circ + \cos^4 45^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Q. If $\sec\theta + \tan\theta = 7$, then evaluate $\sec\theta - \tan\theta$.

[CBSE 2017]

Solution

We know that,

$$sec^2\theta - \tan^2\theta = 1$$

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

$$(7)(\sec\theta - \tan\theta) = 1$$

$$\ldots\left[\sec\theta + \tan\theta = 7;\ (\text{Given})\right)$$

$$\therefore\ \sec\theta - \tan\theta = \frac{1}{7}$$

Q. If $x = r \sin A \cos C$, $y = r \sin A \sin C$ and $z = r \cos A$, then prove that $x^2 + y^2 + z^2 = r^2$.

[CBSE 2017]

Solution

$x = r\sin A\cos C; \quad y = r\sin A\sin C;$

$z = r\cos A$

Squaring and adding

$$L.H.S.\ x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C$$

$$+ r^2\sin^2 A\sin^2 C + r^3\cos^2 A$$

$$= r^2\sin^2 A\left(\cos^2 C + \sin^2 C\right) + r^2\cos^2 A$$

$$= r^2\sin^2 A + r^2\cos^2 A \quad \ldots\left[\cos^2\theta + \sin^2\theta - 1\right]$$

$$= r^2\left(\sin^2 A + \cos^2 A\right) = r^2 = R.H.S.$$

Q. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

[SQP 2022]

Solution

$\sin\theta + \cos\theta = \sqrt{2}$

now square on both side

$= (\sin\theta + \cos\theta)^2 = \sqrt{2}^2$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = 2$

$= 1 + 2\sin\theta\cos\theta = 2$

$\Rightarrow \sin\theta\cos\theta = 1/2$

now

$\tan\theta + \cot\theta = \sin\theta/\cos\theta + \cos\theta/\sin\theta$

$= (\sin^2\theta + \cos^2\theta)/\sin\theta\cos\theta$

$= 1/(1/2) = 2$

Q. If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

[SQP 2022]

Solution

$$\text{Now, } \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \cot^2\theta$$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Q. Prove that

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\text{cosec}\theta$$

[SQP 2022]

Solution

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{(\tan\theta-1)\left(\tan^2\theta+\tan\theta+1\right)}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{1}{\tan\theta}$$

Solution

$$= \tan\theta + 1 + \cot\theta$$

$$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= 1 + \frac{1}{\sin\theta\cos\theta}$$

$$= 1 + \sec\theta \ \mathrm{cosec}\theta = \mathrm{RHS}$$

Q. Prove that

$$\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\frac{\cos\theta}{1-\sin\theta}$$

[CBSE 2016]

Solution

$$L.H.S. = \frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\sec\theta + \tan\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$\ldots \left[\because \sec^2\theta - \tan^2\theta = 1\right.$$

$$= \frac{\sec\theta + \tan\theta - [(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)]}{(1 - \sec\theta + \tan\theta)}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - \sec\theta + \tan\theta]}{(1 - \sec\theta + \tan\theta)} = \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta}$$

$$= \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)\cos\theta} = \frac{1 - \sin^2\theta}{(1 - \sin\theta)\cos\theta}$$

$$= \frac{\cos^2\theta}{(1 - \sin\theta)\cos\theta} = \frac{\cos\theta}{1 - \sin\theta} = R.H.S.$$

Q. Prove that

If $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ and $\dfrac{x}{a}\sin\theta - \dfrac{y}{b}\cos\theta = 1$,

Prove that $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 2$

[CBSE 2017]

Solution

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \qquad \ldots (i)$$

$$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1 \qquad \ldots (ii)$$

Squaring and adding (i) and (ii)

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = 1^2 + 1^2$$

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\left(\frac{x}{a}\cos\theta\right)\left(\frac{y}{b}\sin\theta\right) + \left(\frac{x^2}{a^2}\sin^2\theta\right) + \left(\frac{y^2}{b^2}\cos^2\theta\right) - 2\left(\frac{x}{a}\sin\theta\right)\left(\frac{y}{b}\cos\theta\right) = 1 + 1$$

$$\frac{x^2}{a^2}\left(\cos^2\theta + \sin^2\theta\right) + \frac{y^2}{b^2}\left(\sin^2\theta + \cos^2\theta\right) = 2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \qquad \ldots \left[\because \sin^2\theta + \cos^2\theta = 1\right.$$

Q. Prove that

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\ \mathrm{cosec}\theta-2\sin\theta\cos\theta$$

[SQP 2022]

Solution

$$\text{LHS: } \frac{\sin\theta^{3}/\cos^{3}\theta}{1+\sin^{2}\theta/\cos^{2}\theta}+\frac{\cos^{3}\theta/\sin^{3}\theta}{1+\cos^{2}\theta/\sin^{2}\theta}$$

$$=\frac{\sin^{3}\theta/\cos^{3}\theta}{\left(\cos^{2}\theta+\sin^{2}\theta\right)/\cos^{2}\theta}+\frac{\cos^{3}\theta/\sin^{3}\theta}{\left(\sin^{2}\theta+\cos^{2}\theta\right)/\sin^{2}\theta}$$

$$=\frac{\sin^{3}\theta}{\cos\theta}+\frac{\cos^{3}\theta}{\sin\theta}$$

$$=\frac{\sin^{4}\theta+\cos^{4}\theta}{\cos\theta\,\sin\theta}$$

$$=\frac{\left(\sin^{2}\theta+\cos^{2}\theta\right)^{2}-2\sin^{2}\theta\cos^{2}\theta}{\cos\theta\,\sin\theta}$$

$$=\frac{1-2\sin^{2}\theta\cos^{2}\theta}{\cos\theta\sin\theta}$$

$$=\frac{1}{\cos\theta\,\sin\theta}-\frac{2\sin^{2}\theta\cos^{2}\theta}{\cos\theta\,\sin\theta}$$

$$=\sec\theta\,\mathrm{cosec}\theta-2\sin\theta\cos\theta$$

$$=RHS$$

Q. If $x \sin^3 + y \cos^3 \theta = \sin\theta \cos\theta$, and $x \sin\theta = y \cos\theta$, then prove that $x^2 + y^2 = 1$.

[CBSE 2011]

Solution

$x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$

$\ldots \left[x \sin \theta = y \cos \theta; \frac{x \sin \theta}{\cos \theta} = y \ldots (i) \right.$

$\Rightarrow x \sin^3 \theta + x \frac{\sin \theta}{\cos \theta} \times \cos^3 \theta = \sin \theta \cos \theta$

$\ldots \left[\text{From } (i) \right.$

$\Rightarrow \ x \sin^3 \theta + x \sin \theta \cos^2 \theta = \sin \theta \cos \theta$

$\Rightarrow x \sin \theta \left(\sin^2 \theta + \cos^2 \theta \right) = \sin \theta \cos \theta$

$\Rightarrow x \sin \theta \cdot 1 = \sin \theta \cos \theta$

$\Rightarrow x = \frac{\sin \theta \cos \theta}{\sin \theta} \Rightarrow \ x = \cos \theta$

Put value of x in $(i), y = \cos \theta \cdot \frac{\sin \theta}{\cos \theta}$

As per question, $x^2 + y^2 = 1$

L.H.S. $= x^2 + y^2$

$= sin^2 \theta + \cos^2 \theta = 1 = R.H.S.$

Hence L.H.S = R.H.S

Q. Prove that

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta.$$

[CBSE 2012]

Solution

$$L.H.S.=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$=\frac{\left[\sqrt{1+\sin\theta}\right]^2+\left[\sqrt{1-\sin\theta}\right]^2}{\sqrt{1-\sin\theta}\sqrt{1+\sin\theta}}$$

$$=\frac{1+\sin\theta+1-\sin\theta}{\sqrt{1-\sin^2\theta}}=\frac{2}{\sqrt{\cos^2\theta}}$$

$$=\frac{2}{\cos\theta}=2\times\frac{1}{\cos\theta}$$

$$=2\sec\theta=R.H.S.$$

Q. Prove that

$$\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{\sin^2 A-\cos^2 A}$$

[CBSE 2012]

Solution

$$L.H.S=\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}$$

$$=\frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{(\sin A-\cos A)(\sin A+\cos A)}$$

$$=\frac{\sin^2 A+\cos^2 A+2\sin A\cos A+\sin^2 A+\cos^2 A-2\sin A\cos A}{\sin^2 A-\cos^2 A}$$

$$=\frac{2\left(\sin^2 A+\cos^2 A\right)}{\sin^2 A-\cos^2 A}$$

$$=\frac{2(1)}{\sin^2 A-\cos^2 A}\quad\ldots\left[\because \sin^2 A+\cos^2 A=1\right]$$

$$=\frac{2}{\sin^2 A-\cos^2 A}=R.H.S$$

Q. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \csc\theta = n$, then prove that $n(m^2 - 1) = 2m$.

[CBSE 2013]

Solution

$$m^2-1=(\sin\theta+\cos\theta)^2-1$$
$$=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1$$
$$=1+2\sin\theta\cos\theta-1$$
$$=2\sin\theta\ \cos\theta\quad\ldots\left[\sin^2\theta+\cos^2\theta=1\right.$$

$$LHS=n\left(m^2-1\right)$$
$$=\left(\sec\theta+\operatorname{cosec}\theta\right)2\sin\theta\cos\theta$$
$$=\left[\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right]2\sin\theta\cos\theta$$
$$=\left[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\right]2\sin\theta\cos\theta$$
$$=2\left[\sin\theta+\cos\theta\right]$$
$$=2m=RHS\quad\ldots\left[\because\ \sin\theta+\cos\theta=m\right.$$

Q. Find the value of

$$\frac{\sin\theta}{(\sec\theta+\tan\theta-1)}+\frac{\cos\theta}{(\text{cosec}\theta+\cot\theta-1)}.$$

[CBSE 2013]

Solution

$$\frac{\sin\theta}{\sec\theta+\tan\theta-1}+\frac{\cos\theta}{\cos ec\theta+\cot\theta-1}$$

$$=\frac{\sin\theta}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-1}+\frac{\cos\theta}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-1}$$

$$=\frac{\sin\theta}{\frac{1+\sin\theta-\cos\theta}{\cos\theta}}+\frac{\cos\theta}{\frac{1+\cos\theta-\sin\theta}{\sin\theta}}$$

$$=\frac{\sin\theta\cos\theta}{1+\sin\theta-\cos\theta}+\frac{\sin\theta\cos\theta}{1+\cos\theta-\sin\theta}$$

$$=\sin\theta\cos\theta\left(\frac{1}{1+\sin\theta-\cos\theta}+\frac{1}{1+\cos\theta-\sin\theta}\right)$$

$$=\sin\theta\cos\theta\left(\frac{1+\cos\theta-\sin\theta+1+\sin\theta-\cos\theta}{(1+\sin\theta-\cos\theta)(1-\sin\theta+\cos\theta)}\right)$$

$$=\sin\theta\cos\theta\left(\frac{2}{[1+(\sin\theta-\cos\theta)][1-(\sin\theta-\cos\theta)}\right)$$

Solution

$$= \sin\theta\cos\theta\left(\frac{2}{(1)^2-(\sin\theta-\cos\theta)^2}\right)$$

$$= \sin\theta\cos\theta\left(\frac{2}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}\right)$$

$$= \sin\theta\cos\theta\left(\frac{2}{1-(1-2\sin\theta\cos\theta)}\right)$$

$$\ldots\left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$= \sin\theta\cos\theta\left(\frac{2}{2\sin\theta\cos\theta}\right)=1$$

Q. Prove that :

$(\cot A + \sec B)^2 - (\tan B - \text{cosec } A)^2 =$
$2(\cot A \cdot \sec B + \tan B \cdot \text{cosec } A)$

[CBSE 2014]

Solution

$$\text{LHS} = (\cot A + \sec B)^2 - (\tan B - \cos ec A)^2$$

$$= \cot^2 A + \sec^2 B + 2\cot A \sec B$$

$$- \left(\tan^2 B + \cos ec^2 A - 2\tan B \cos ec A\right)$$

$$= \cot^2 A + \sec^2 B + 2\cot A \sec B - \tan^2 B$$

$$- \cos ec^2 A + 2\tan B \ \cos ec A$$

$$= \left(\sec^2 B - \tan^2 B\right) - \left(\cos ec^2 A - \cot^2 A\right)$$

$$+2\left(\cot A \sec B + \tan B \cos ec A\right)$$

$$= 1 - 1 + 2\left(\cot A \sec B + \tan B \cos ec A\right)$$

$$\ldots \left[\because \sec^2 B - \tan^2 B = 1 \cos ec^2 A - \cot^2 A = 1\right]$$

$$= 2\left(\cot A . \sec B + \tan B . \cos ec A\right)$$

$$= RHS$$

Q. Prove that:

$(\sin\theta + \cos\theta + 1).(\sin\theta - 1 + \cos\theta).\sec\theta.\mathrm{cosec}\,\theta = 2$

[CBSE 2014]

Solution

$$LHS = (\sin\theta + \cos\theta + 1)$$

$$(\sin\theta - 1 + \cos\theta).\sec\theta\cos ec\theta$$

$$= [(\sin\theta + \cos\theta) + 1]$$

$$[(\sin\theta + \cos\theta) - 1].\sec\theta\cos ec\theta$$

$$= \left[(\sin\theta + \cos\theta)^2 - (1)^2\right]\sec\theta\cos ec\theta$$

$$\ldots\left[\because (a+b)(a-b) = a^2 b^2\right]$$

$$= \left[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right].\sec\theta\cos ec\theta$$

$$= (1 + 2\sin\theta\cos\theta - 1).\sec\theta\cos ec\theta$$

$$\ldots\left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= (2\sin\theta\cos\theta).\frac{1}{\cos\theta}.\frac{1}{\sin\theta}$$

$$= 2 = RHS$$

Q. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, If;

(i) $x = a \sec \theta$, $y = b \tan \theta$

(ii) $x = a \operatorname{cosec} \theta$, $y = b \cot \theta$

[CBSE 2014]

Solution

$$(i) LHS = b^2x^2 - a^2y^2$$

$$= b^2(a\ \sec\theta)^2 - a^2(b\tan\theta)^2$$

$$= b^2a^2\sec^2\theta - a^2b^2\tan^2\theta$$

$$= b^2a^2(\sec^2\theta - \tan^2\theta)$$

$$= b^2a^2(1) = a^2b^2 = RHS \ldots [\because \sec^2\theta - \tan^2\theta = 1]$$

$$(ii) LHS = b^2x^2 - a^2y^2$$

$$= b^2(a\ \cos ec\theta)^2 - a^2(b\ \cot\theta)^2$$

$$= b^2a^2\cos ec^2\theta - a^2b^2\cot^2\theta$$

$$= b^2a^2(\cos ec^2\theta - \cot^2\theta)$$

$$= b^2a^2(1) \quad \ldots . [\because \cos ec^2\theta - \cot^2\theta = 1]$$

$$= a^2b^2 = RHS$$

Q. Prove that

$$(1+\cot A+\tan A).(\sin A-\cos A)=\frac{\sec^3 A-\operatorname{cosec}^3 A}{\sec^2 A\cdot\operatorname{cosec}^2 A}.$$

[CBSE 2015]

Solution

$$LHS = (1 + \cot A + \tan A)(\sin A - \cos A)$$

$$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$$

$$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A}\right)(\sin A - \cos A)$$

$$\ldots \left[\text{Using } a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)\right.$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$$

$$= \frac{\frac{\sin^3 A}{\sin^3 A \cos^3 A} - \frac{\cos^3 A}{\sin^3 A \cos^3 A}}{\frac{\sin A \cos A}{\sin^3 A \cos^3 A}}$$

$$\ldots \left[\text{dividing Num. \& Denom. by } sin^3 A \cdot \cos^3 A\right.$$

$$= \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A \cdot \mathrm{cosec}^2 A} = RHS \ (\text{Hence proved})$$

Q. If $x = a \cos \theta - b \sin \theta$ and
$y = a \sin \theta + b \cos \theta$,
then prove that $a^2 + b^2 = x^2 + y^2$.

[CBSE 2015]

Solution

RHS = $x^2 + y^2$

$= (a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2$

$= a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \cos \theta \sin \theta +$
$a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta$

$= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2\theta)$

$= a^2(\cos^2 \theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2 \theta)$

$= a^2 + b^2 =$ LHS ...[$\because \cos^2\theta + \sin^2 \theta = 1$]

Q. Prove the following:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$$

Solution

$$\Rightarrow \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$\cot\theta=\frac{1}{\tan\theta}$$

$$\Rightarrow \frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta\left(\tan\theta-1\right)}$$

$$\Rightarrow \frac{\tan^3\theta-1}{\tan\theta\left(\tan\theta-1\right)}$$

$$a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)$$

$$\Rightarrow \frac{\left(\tan\theta-1\right)\left(\tan^2\theta+\tan\theta+1\right)}{\tan\theta\left(\tan\theta-1\right)}$$

$$\Rightarrow \frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$$

$$\Rightarrow \tan\theta+1+\cot\theta=1+\tan\theta+\cot\theta\left(RHS\right)$$

Q. In Δ PQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Solution

$\Rightarrow PQ = 5cm$

$\Rightarrow PR + QR = 25cm$

$\Rightarrow PR = 25 - QR$

Now, In ΔPQR

$\Rightarrow (PR)^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = 5^2 + QR^2$

$\Rightarrow 625 + QR^2 - 50QR = 25 + QR^2$

$\Rightarrow 50QR = 600$

$\Rightarrow QR = 12cm$

$\Rightarrow PR = 25 - 12 = 13cm$

$$\therefore \sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13}, \tan P = \frac{QR}{PQ} = \frac{12}{5}$$

Hence, the answers are $\sin P = \frac{12}{13}, \cos P = \frac{5}{13}, \tan P = \frac{12}{5}$.

Q. If $\sec\theta + \tan\theta = p$, prove that

$$\sin\theta = \frac{p^2-1}{p^2+1}$$

Solution

$$R.H.S. = \frac{p^2 - 1}{p^2 + 1}$$

$$= \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + (1 + \tan^2\theta) + 2\sec\theta\tan\theta}$$

$$\ldots\left[\begin{array}{l} \because sec^2\theta - 1 = \tan^2\theta \\ \sec^2\theta = 1 + \tan^2\theta \end{array}\right.$$

Solution

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta\,(\tan\theta + \sec\theta)}{2\sec\theta\,(\sec\theta + \tan\theta)} = \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \sin\theta = L.H.S. \quad \ldots \text{(Hence proved)}$$

Q. Evaluate the following:

$$\frac{2\left(\cos^2 45^\circ + \tan^2 60^\circ\right) - 6\left(\sin^2 45^\circ - \tan^2 60^\circ\right)}{\tan 30^\circ + \cot 60^\circ}$$

Solution

$$\frac{2\left(\cos^2 45^\circ + \tan^2 60^\circ\right) - 6\left(\sin^2 45^\circ - \tan^2 60^\circ\right)}{\tan 30^\circ + \cot 60^\circ}$$

$$= \frac{2\left(\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2\right) - 6\left(\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\sqrt{3}\right)^2\right)}{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$$

$$= \frac{2\left(\frac{1}{2} + 3\right) - 6\left(\frac{1}{2} - 3\right)}{\frac{2}{\sqrt{3}}}$$

$$= \left[2\left(\frac{1+6}{2}\right) - 6\left(\frac{1-6}{2}\right)\right] \times \frac{\sqrt{3}}{2}$$

$$= (7 + 15)\frac{\sqrt{3}}{2} = 22 \times \frac{\sqrt{3}}{2} = 11\sqrt{3}$$

Q. Prove that

$$\frac{1+\cos A}{\sin A}+\frac{\sin A}{1+\cos A}=2\operatorname{cosec} A$$

Solution

$$L.H.S. = \frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$$

$$= \frac{(1+\cos A)(1+\cos A) + \sin A \sin A}{\sin A(1+\cos A)}$$

$$= \frac{(1+\cos A)^2 + \sin^2 A}{(1+\cos A)\sin A}$$

$$= \frac{1+\cos^2 A + 2\cos A + \sin^2 A}{(1+\cos A)\sin A}$$

$$= \frac{1+2\cos A + 1}{(1+\cos A)\sin A}$$

$$= \frac{2+2\cos A}{(1+\cos A)\sin A}$$

Solution

$$= \frac{2\left(1+\cos A\right)}{\left(1+\cos A\right)\sin A}$$

$$= 2\frac{1}{\sin A}$$

$$= 2\text{cosec } A$$

$$= R.H.S$$

Hence proved.

Q. If $x = \sec\theta + q\tan\theta$ and
$y = p\tan\theta + q\sec\theta$, then prove that
$x^2 - y^2 = p^2 - Q$.

Solution

L.H.S. $= x^2 - y^2$

$= (p \sec \theta + q \tan \theta)^2 - (p \tan \theta + q \sec \theta)^2$

$= p^2 \sec^2 \theta + q^2 \tan^2 \theta + 2\ pq \sec\theta\tan\theta$

$-(p^2 \tan^2 \theta + q^2 \sec^2 \theta + 2pq \sec \theta \tan \theta)$

$= p^2 \sec^2 \theta + q^2 \tan^2 \theta + 2pq \sec\theta \tan\theta$

$- p^2 \tan^2 \theta - q^2 \sec^2 \theta - 2pq \sec \theta \tan \theta$

$= p^2(\sec^2\theta - \tan^2\theta) - q^2(\sec^2 \theta - \tan^2\theta)$

$= p^2 - q^2$ ...[$\sec^2 \theta - \tan^2 \theta = 1$

= R.H.S.

Q. Prove that:

$$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{2}{2\sin^2\theta-1}$$

Solution

$$L.H.S. = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin\theta^2 + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{1+1}{\sin^2\theta - (1 - \sin^2\theta)} = \frac{2}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1} = R.H.S. \quad \ldots \text{(Hence proved)}$$

Q. If $\tan\theta = \dfrac{a}{b}$, then prove that

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

Solution

$$\text{L.H.S.} = \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$$

$$= \frac{\frac{a\sin\theta}{\cos\theta} - \frac{b\cos\theta}{\cos\theta}}{\frac{a\sin\theta}{\cos\theta} + \frac{b\cos\theta}{\cos\theta}} = \frac{a\tan\theta - b}{a\tan\theta + b}$$

$\ldots$ [Dividing num. and deno. by cos θ

$$= \frac{a\left(\frac{a}{b}\right) - b}{a\left(\frac{a}{b}\right) + b} \quad \ldots\left[\because \tan\theta = \frac{a}{b} \quad \ldots [\text{Given}\right.$$

$$= \frac{\frac{a^2}{b} - b}{\frac{a^2}{b} + b} = \frac{\frac{a^2-b^2}{b}}{\frac{a^2+b^2}{b}} = \frac{a^2 - b^2}{a^2 + b^2}$$

$$= R.H.S.$$

Q. A man goes 15 m due west and then 8 m due north. Find the distance of the man from the starting point.

[CBSE 2016]

Solution

The man begins from O andgoes to A and then to B aking rt. angle ΔOAB.

$OB^2 = OA^2 + AB^2$ ... [Pythagoras' theorem

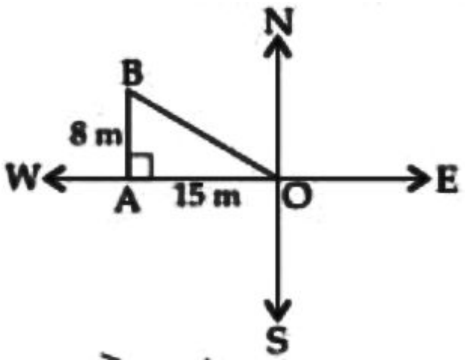
$= 15^2 + 8^2$

$= 225 + 64$

$= 289$

$\therefore \; OB = +\sqrt{289}$

$= 17m$



Q. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

[CBSE 2016]

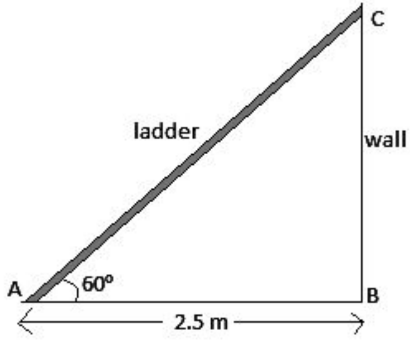
Solution

Let AC be the ladder.

$$\cos 60^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$

$\therefore$ Length of ladder, AC $= 5m$

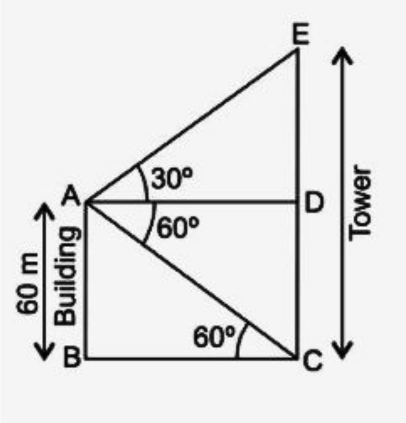


Q. The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 m high, are 30° and 60° respectively. Find the difference between heights of the building and the tower and the distance between them.

[CBSE 2014]

Solution

Let AB $(60m)$ be the building and DCE be the tower. In rt. ΔABC, $\tan 60^0 = \frac{AB}{BC}$



$$\sqrt{3} = \frac{60}{BC}$$

$$\sqrt{3}BC = 60$$

$$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{60\sqrt{3}}{3} = 20\sqrt{3}\ m \quad \dots (i)$$

$$AD = BC = 20\sqrt{3}m$$

In rt. ΔADE, $\tan 30^0 = \frac{DE}{AD}$

$$\frac{1}{\sqrt{3}} = \frac{DE}{20\sqrt{3}} \Rightarrow \sqrt{3}DE = 20\sqrt{3}$$

$$DE = \frac{20\sqrt{3}}{\sqrt{3}} = 20m$$

$\therefore$ Difference between the heights of the building and the tower DE $=$ **20m**

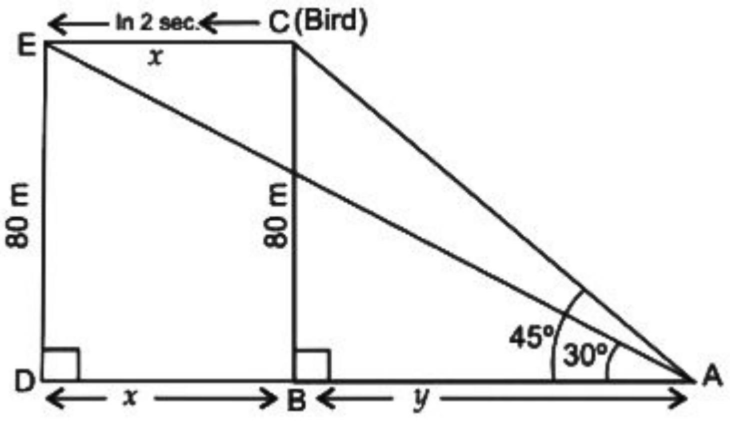
Distance between them,

$$BC = 20\sqrt{3}m = 20\,(1.73) = 34.6\ m$$

Q. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of the flying bird. $\left(\text{Take } \sqrt{3} = 1.732\right)$

[CBSE 2016]

Solution



Let BC be the tree and BD & AB are x and y respectively.

In rt. $\Delta ABC, \tan 45^\circ = \dfrac{BC}{AB}$

$$\Rightarrow 1 = \frac{80}{y} \Rightarrow y = 80m \quad \ldots (i)$$

Solution

In rt. $\Delta ADE, \tan 30^\circ = \dfrac{DE}{AD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

$$\Rightarrow x + y = 80\sqrt{3}$$

$$\Rightarrow x + 80 = 80\sqrt{3} \quad \ldots [\text{From (i)}]$$

$$\Rightarrow x = 80\sqrt{3} - 80$$

$$\Rightarrow x = 80\left(\sqrt{3} - 1\right)$$

$$\Rightarrow x = 80\,(1.732 - 1)$$

$$\ldots \left(\sqrt{3} = 1.732\right)$$

$$\Rightarrow x = 80\,(0.732)$$

$$\therefore \text{CE}, x = 58.56m$$

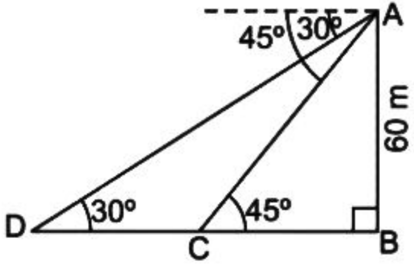
Hence, speed of bird $= \dfrac{\text{Distance}}{\text{Time}}$

$$= \frac{\text{CE}}{\text{Time}} = \frac{58.56m}{2\text{ sec.}} = 29.28\text{ m/sec.}$$

Q. As observed from the top of a 60 m high light-house from the sea-level, the angles of depression of two ships are 30^0 and 45^0. If one ship is exactly behind the other on the same side of the light-house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

[CBSE 2013]

Solution



Let $AB = 60\ m$ be the Lighthouse, and C and D

In right ΔABC

$\tan 45^0 = \frac{AB}{BC}$

$1 = \frac{60}{BC}$

$BC = 60\ m \quad \ldots (i)$

In right ΔABD, $\tan 30^0 = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{60}{BD} \Rightarrow BD = 60\sqrt{3}$

$\Rightarrow BD = 60\,(1.732) = 103.92\ m$

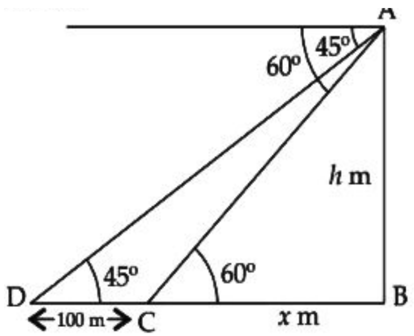
$\therefore$ Distance between the two ships,

$CD = BD - BC = 103.92 = 43.92m.$

Q. From the top of a vertical tower, the angles of depression of two cars in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use √ = 1.73]

[CBSE 2011]

Solution



Let AB be the tower

Let AB = $h\ m$, and BC = x m

In rt. ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3}x = h \qquad \Rightarrow x = \frac{h}{\sqrt{3}} \qquad \ldots (i)$$

In rt. ΔABD,

$$\tan 45^\circ = \frac{AB}{BD} \qquad \Rightarrow 1 = \frac{h}{x+100}$$

$$\Rightarrow h = x + 100 \qquad \Rightarrow h = \frac{h}{\sqrt{3}} + 100$$

$$\ldots \text{[From } (i)$$

$$\Rightarrow h = \frac{h+100\sqrt{3}}{\sqrt{3}} \Rightarrow \sqrt{3}h = h + 100\sqrt{3}$$

$$\Rightarrow \sqrt{3}h - h = 100\sqrt{3}$$

$$\Rightarrow h\left(\sqrt{3}-1\right) = 100\sqrt{3}$$

$$h = \frac{100\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{100\left(3+\sqrt{3}\right)}{3-1}$$

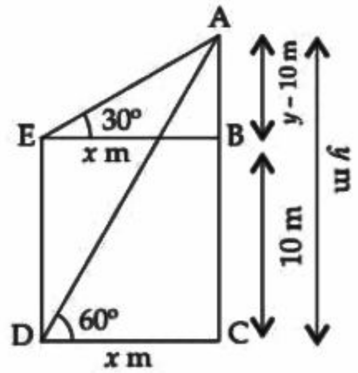
$$= 50\,(3 + 1.73) = 50\,(4.73) = 236.5$$

$\therefore$ Height of the tower, $h = 236.5m$

Q. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 30°. Find the height of the tower.

[CBSE 2011]

Solution



Let AC be the tower

Let $AC = ym$

Let DC = EB = xm

In rt. ΔABE,

$$tan30^\circ = \frac{AB}{BE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y-10}{x}$$

$$\Rightarrow x = \sqrt{3}\,(y-10) \qquad \ldots (i)$$

$$\tan 60^\circ = \frac{AC}{DC}$$

$$\Rightarrow \frac{\sqrt{3}}{1} = \frac{y}{x} \qquad \Rightarrow \sqrt{3}x = y$$

$$\sqrt{3}\cdot\sqrt{3}\,(y-10) = y \qquad \ldots [\text{From } (i)$$

$$3\,(y-10) = y \qquad \Rightarrow \qquad 3y - 30 = y$$

$$3y - y = 30 \qquad \Rightarrow \qquad 2y = 30$$

$\therefore$ Height of the tower, $y = 15\,m$

Q. From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 30° and 45°. Find the height of the hill. [Use √ = 1.73]

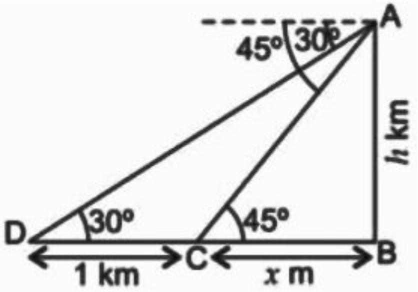
[CBSE 2012]

Solution

Let AB be the hill of height h km.

Let C and D be two stones due east of the hill at a distance of 1 km from each other.

Let $BC = x\ km$



In rt. ΔABC,

$$tan45^\circ = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\Rightarrow h = x \quad \dots . (i)$$

In rt. $\Delta ABD, tan30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

$$\sqrt{3}h = x + 1$$

$$\sqrt{3}x = x + 1 \qquad \dots [\text{From } (i)$$

Solution

$$\sqrt{3}x - x = 1$$

$$(\sqrt{3} - 1)x = 1$$

$$x = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$x = \frac{\sqrt{3}+1}{3-1}$$

$$x = \frac{1.73+1}{2} \qquad \ldots [\because \sqrt{3} = 1.73$$

$$x = \frac{2.73}{2} = 1.365\ km$$

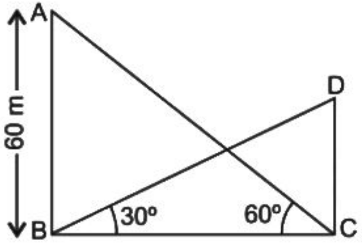
Hence, the height of the hill is 1.365 km.

Q. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

[CBSE 2013]

Solution

Let $AB = 60m$ be the tower and Let CD be the building



In ight ΔABC,

$\tan 60^0 = \frac{AB}{BC}$

$\sqrt{3} = \frac{60}{BC} \Rightarrow \sqrt{3}BC = 60$

$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}m \ldots (i)$

In right ΔBCD, $\tan 30^0 = \frac{CD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{CD}{20\sqrt{3}}$

$\sqrt{3}CD = 20\sqrt{3} \quad \therefore CD = 20\ m$

$\therefore$ Height of the building, CD $= 20$m

Q. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

Solution

Let A be the point on the ground and C be the aeroplane

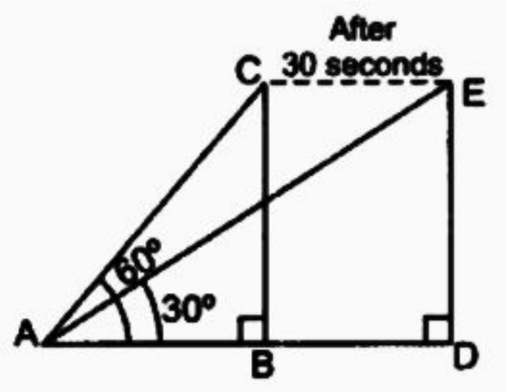
In rt. ΔABC,

$$\tan 60^\circ = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{AB}$$

$$\Rightarrow \quad \sqrt{3}AB = 3000\sqrt{3}$$

$$AB = 3000m$$



Solution

In rt. ΔADE, $tan30^\circ = \frac{DE}{AD}$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{AD}$$

$AD = 3000 \times 3 = 9000m$

$CE = BD$

$BD = AD - AB$

$= 9,000 - 3,000 = 6,000m$

$\therefore$ Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{6000}{30}$

$= 200 \text{ meter/sec.}$

$= \frac{200}{1000} \times 3,600$

$= 720\ km/hr.$

Q. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars.
[Use $\sqrt{3} = 1.73$]

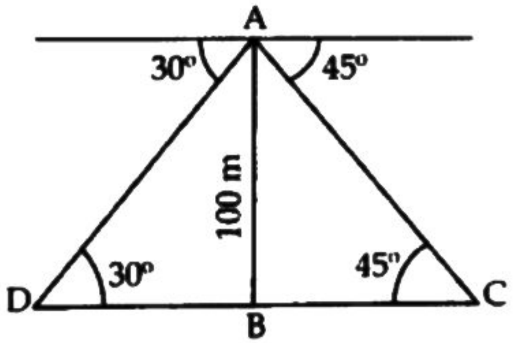
Solution

Let AB be the tower.

In rt. ΔABC, $tan45^\circ = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{100}{BC}$

$\Rightarrow \ BC = 100m$



$In\,rt.\ \Delta ABD,$

$\tan 30^\circ = \frac{AB}{BD}$

$BD = 100\sqrt{3} = 100\,(1.73) = 173\,m$

$\therefore$ Distance between the cars, CD

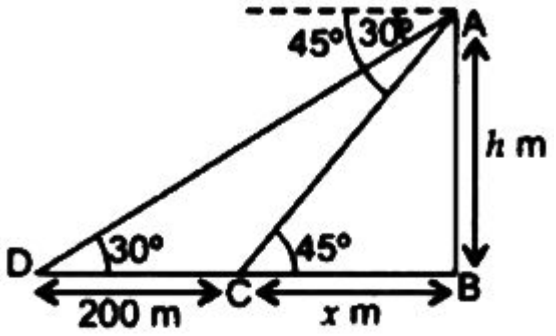
$= BD + BC$

$= 173 + 100$

$= 273\,m$

Q. The angles of depression of two ships from the top of a lighthouse and on the same side of it are found to be 45° and 30°. If the ships are 200 m apart, find the height of the light house. [Use $\sqrt{3} = 1.73$]

Solution



$$\frac{1}{\sqrt{3}} = \frac{h}{x+200}$$

$$\sqrt{3}h = x + 200$$

$$\sqrt{3}h = h + 200 \qquad \ldots[\text{from } (i)$$

$$\sqrt{3}h - h = 200$$

$$\left(\sqrt{3}-1\right)h = 200$$

$$h = \frac{200}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{200\,(1.73+1)}{3-1} = 100\,(2.73)$$

$$\ldots\left[\because \sqrt{3} = 1.73\right.$$

$$h = 273m$$

$\therefore$ Height of the light house $= 273$ m

Q. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

Solution

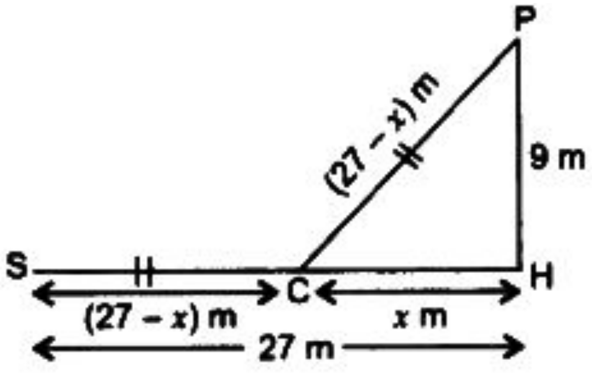
Let PH be the pillar. Let the distance from the hole to the place where snake is caught = x m

Let P be the top of the pillar and S be the point where the snake is

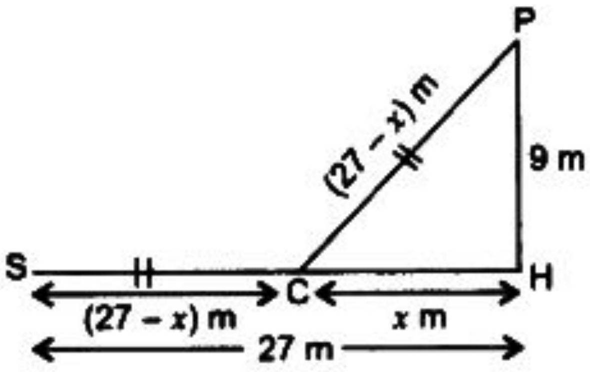
$\therefore$ SC = (27 - x)m

SC = PC = (27 - x)m ...[$\because$ Their speeds are equal

In rt. ΔPHC



Solution



$PH^2 + CH^2 = PC^2 \ldots$ [Pythagoras' theorem

$9^2 + x^2 = (27 - x)^2$

$81 + x^2 = 729 - 54x + x^2$

$54x = 729 - 81 = 648$

$x = \frac{648}{54} = 12m$

Hence, required distance, $x = 12m$

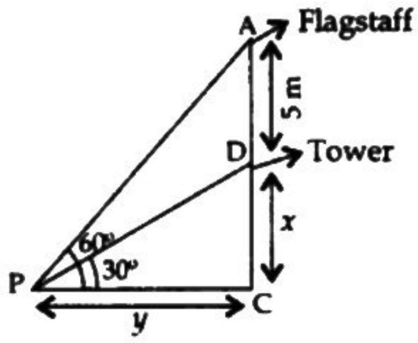
Q. From a point P on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5m, find the height of the tower.

Solution

In $ABCD$,

$$(i)\ \frac{x}{y}=\tan 30^\circ=\frac{1}{\sqrt{3}}$$

$$y=\sqrt{3}x\ \ldots(i)$$



$In\ \Delta ABC,$

$$(ii)\ \frac{x+5}{y}=\tan 60^\circ$$

$$\tan 60^\circ=\sqrt{3}$$

$$\frac{x+5}{\sqrt{3}x}=\sqrt{3}\ \ldots[\text{From }(i)$$

$$\Rightarrow\ 3x=x+5\text{ or }x=2.5$$

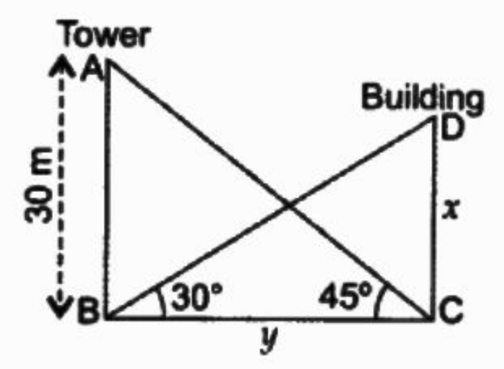
$$\therefore\ \text{Height of Tower }=x=2.5m$$

Q. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Solution

Let height of building be x and the distance between tower and building be y.

In ΔABC,



$$tan 45^\circ = \frac{AB}{BC}$$

$$\frac{30}{y} = \tan 45^\circ = 1$$

$$\Rightarrow \quad y = 30 \qquad \ldots (i)$$

Now, In ΔBCD

$$\tan 30^\circ = \frac{CD}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{y}$$

$$\Rightarrow x = \frac{y}{\sqrt{3}} = \frac{30}{\sqrt{3}} = \frac{30 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \ldots [\text{From } (i)$$

$$\Rightarrow x = \frac{30 \times \sqrt{3}}{3} = 10\sqrt{3}$$

$\therefore$ Height of the building is $10\sqrt{3}\, m$.

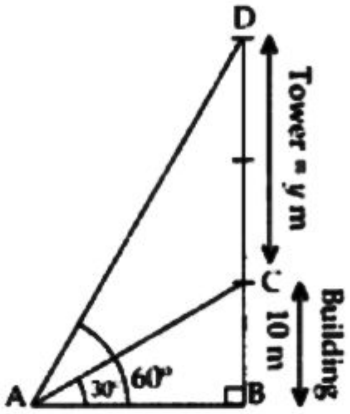
Q. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 10 m high building are 30° and 60° respectively. Find the height of the tower

Solution

Let BC be the building and CD be the transmission tower.

A be the point on the ground.

Let CD = ym



In rt. ΔABC,

$$tan30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AB}$$

$$AB = 10\sqrt{3}m \qquad \ldots(i)$$

$$Inrt.\ \Delta ABD,\ \tan 60^\circ = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{10+y}{10\sqrt{3}} \qquad \ldots[\text{From } (i)$$

$$\Rightarrow \ 10+y=30 \qquad \Rightarrow y = 30-10 = 20$$

$$\therefore \text{ Height of transmission tower } = 20m$$

Q. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y.

Solution

When base is same for both towers and their heights are given, i.e., x and y respectively

Let the base of towers be k.

$$tan30^\circ = \frac{x}{k}, \qquad \tan 60^\circ = \frac{y}{k}$$

$$x = k\tan 30^\circ = \frac{k}{\sqrt{3}} \ldots (i) \quad y = k\tan 60^\circ = k\sqrt{3} \ldots (ii)$$

From equation (i) and (ii),

$$\frac{x}{y} = \frac{\frac{k}{\sqrt{3}}}{k\sqrt{3}} = \frac{k}{\sqrt{3}} \times \frac{1}{k\sqrt{3}} = \frac{1}{3} = 1:3$$

Mensuration

Q. If a circular grass lawn of 35 m in radius has a path 7 m wide running around it on the outside, then the area of the path is

[Board 2022 Term 1 SQP STD]

Solution

Radius of outer concentric circle,

$$= (35 + 7)\,\text{m} = 42\text{ m.}$$

$$\text{Area of path} = \pi(42^2 - 35^2)\,\text{m}^2$$

$$= \frac{22}{7}(42^2 - 35^2)\,\text{m}^2$$

Thus (c) is correct option.

Q. The area of the circle that can be inscribed in a square of side 6 cm is

[Board 2022 Term 1 STD]

Solution

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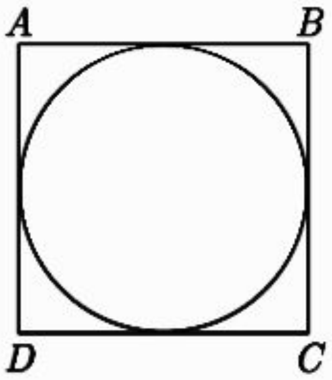


Diameter of circle is equal to the side of square.

Diameter of a circle, $d = = 6 \text{ cm}$

Radius of a circle, $r = \frac{d}{2} = \frac{6}{2} = 3 \text{ cm}$

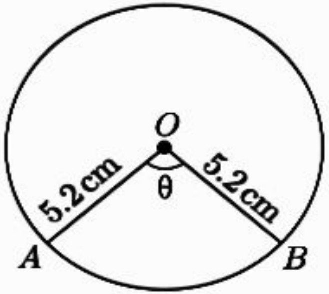
Area of circle, $\pi r^2 = \pi 3^2 = 9\pi \text{ cm}^2$

Thus (d) is correct option.

Q. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

[[Board 2020 Delhi Standard]]

Solution



Perimeter of the sector

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$16.4 = 10.4 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$6 = \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$\frac{3}{5.2} = \frac{\theta \times \pi}{360^\circ}$$

Now, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \left(\frac{\theta \times \pi}{360^\circ}\right) r^2$

$$= \frac{3}{5.2} \times (5.2)^2 = 15.6 \text{ sq. units.}$$

Q. If the perimeter of a protractor is 72 cm, calculate its area. Use $\pi = 22/7$.

[Board Term-2 OD 2012]

Solution

Perimeter of semi-circle

$$\pi r + 2r = 72 \text{ cm}$$

$$(\pi + 2)r = 72 \text{ cm}$$

$$\left(\frac{22}{7} + 2\right)r = 72 \text{ cm}$$

$$r\left(\frac{22+14}{7}\right) = 72 \text{ cm}$$

$$\frac{36}{7}r = 72 \Rightarrow r = 14 \text{ cm}$$

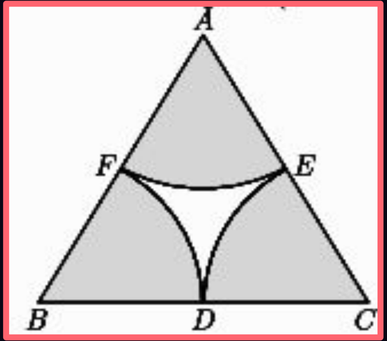
Area of protractor,

$$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$

$$= 308 \text{ cm}^2$$

Q. In given figure arcs are drawn by taking vertices A B, and C of an equilateral triangle of side 10 cm, to intersect the side BC , CA and AB at their respective mid-points D E, and F . Find the area of the shaded region. (Use π = 3 1. 4).

[Board Term-2 2011]



Solution

Since ΔABC is an equilateral triangle

$$\angle A = \angle B = \angle C = 60^\circ$$

Here we have 3 sector and area of all three sector is equal.

Area of sector $AFEA$,

$$\text{Area}_{AFEA} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \pi (5)^2 = \frac{25}{6}\pi \text{ cm}^2$$

Thus total area of shaded region

$$\text{Area} = 3\left(\frac{25}{6}\pi\right) = \frac{25 \times 3.14}{2}$$

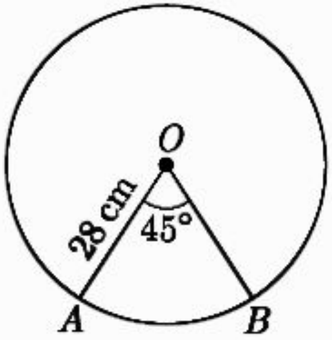
$$= 39.25 \text{ cm}^2$$

Q. Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle 45^0.

[Board Term-2 2015]

Solution

As per question statement figure is shown below;



Area of major sector,

$$= \text{area of circle} - \text{area of minor sector}$$

$$= \pi r^2\left(1 - \frac{\theta}{360^\circ}\right)$$

$$= \frac{22}{7} \times 28 \times 28\left(1 - \frac{45^\circ}{360^\circ}\right)$$

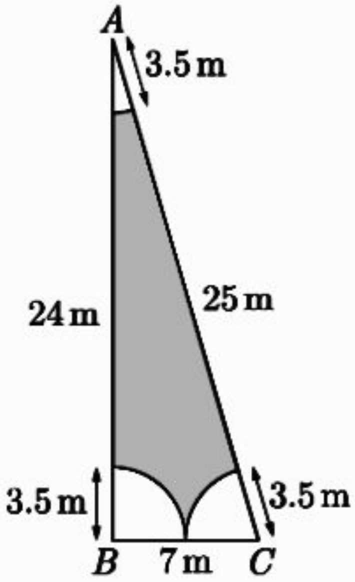
$$= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8}$$

$$= 2156 \text{ cm}^2$$

Q. Sides of a right triangular field are 25 m, 24 m and 7 m. At the three corners of the field, a cow, a buffalo and a horse are tied separately with ropes of 3.5 m each to graze in the field. Find the area of the field that cannot be grazed by these animals.

[Board 2020 SQP Standard]

Solution



Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the animals is the sum of the areas of three sectors with central angles θ_1, θ_2 and θ_3 each with radius $r = 3.5$ m.

$$\frac{\pi r^2 \theta_1}{360^\circ} + \frac{\pi r^2 \theta_2}{360^\circ} + \frac{\pi r^2 \theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1 + \theta_2 + \theta_3) \qquad \text{...(1)}$$

Solution

Substituting above in equation (1) we have

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ}=\frac{\pi r^2}{360^\circ}\times 180^\circ=\frac{\pi r^2}{2}$$

$$=\frac{22}{7}\times\frac{1}{2}\times(3.5)^2$$

$$=19.25$$

Hence, the area grazed by the horses is $19.25\ \text{m}^2$.

$$\text{Area of }\Delta\,ABC=\frac{1}{2}\times AB\times BC$$

$$=\frac{1}{2}\times 24\times 7=84\ \text{m}^2$$

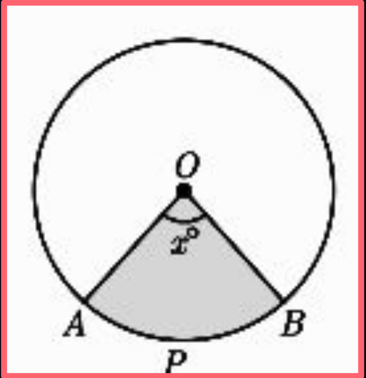
Area of the field that cannot be grazed by these animals

= Area of triangle − Area of three sectors

$$=84-1925=64.75\ \text{m}^2$$

Q. In given fig., O is the centre of a circle. If the area of the sector OAPB is 5/36 times the area of the circle, then find the value of *x*.

[Board Term-2 2012]



Solution

Area of the sector,

$$A_{\frown} = \frac{\pi r^2 \theta}{360^\circ}$$

Area of sector $OAPB$ is $\frac{5}{36}$ times the area of circle.

Thus

$$\pi r^2 \times \frac{x}{360} = \frac{5}{36} \pi r^2$$

$$\frac{x}{360} = \frac{5}{36}$$

$$x = 50^\circ$$

Q. The area of a circular play ground is 22176 cm^2 . Find the cost of fencing this ground at the rate of 50 per metre.

[Board 2020 OD Standard]

Solution

Area of a circular play ground,

$$A = 22176 \text{ cm}^2$$

i.e., $$\pi r^2 = 22176 \text{ cm}^2$$

$$r^2 = 22176 \times \frac{7}{22}$$

$$= 7056$$

$$r = 84 \text{ cm} = 0.84 \text{ m}$$

Perimeter of ground,

$$p = 2\pi r$$

Cost of fencing this ground,

$$= ₹\,50 \times 2\pi r$$

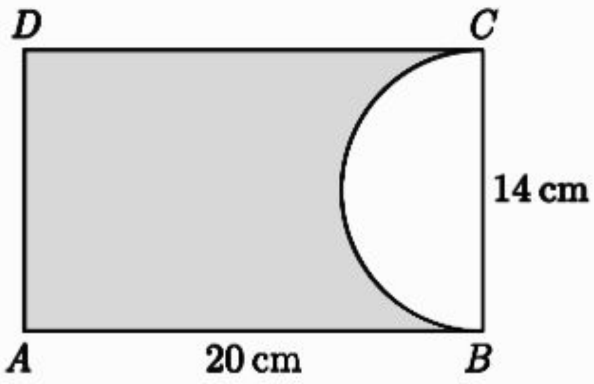
$$= ₹\,50 \times 2 \times \frac{22}{7} \times 0.84 = ₹\,264$$

Q. A paper is in the form of a rectangle ABCD in which AB = 20 cm, BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of the part. Use π =22/7.

[Board Term-2 2012, Foreign 2014]

Solution

As per question the digram is shown below.



Area of remaining part,

$$= \text{Area of rectangle} - \text{Area of semi-circle}$$

$$= 20 \times 14 - \frac{1}{2}\pi 7^2$$

$$= 280 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= 280 - 77 = 203 \text{ cm}$$

Hence area of remaining part is 203 cm

Q. If the difference between the circumference and the radius of a circle is 37 cm, then using π =22/7, find the circumference (in cm) of the circle

[Board Term-2 Delhi 2012]

Solution

Let r be the radius of the circle.

Now, circumference − radius = 37

$$2\pi r - r = 37$$

$$2 \times \frac{22}{7} r - r = 37$$

$$r\left(\frac{22-7}{7}\right) = 37$$

$$r \times \frac{37}{7} = 37$$

$$r = \frac{37 \times 7}{37} = 7 \text{ cm}$$

Circumference of the circle,

$$2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

Q. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of sector formed by the arc.

[Board Term-2 Delhi Compt. 2017]

Solution

We have $r = 21$ cm and $\theta = 60^\circ$

Area formed the sector $= \frac{\theta}{360} \times \pi r^2$

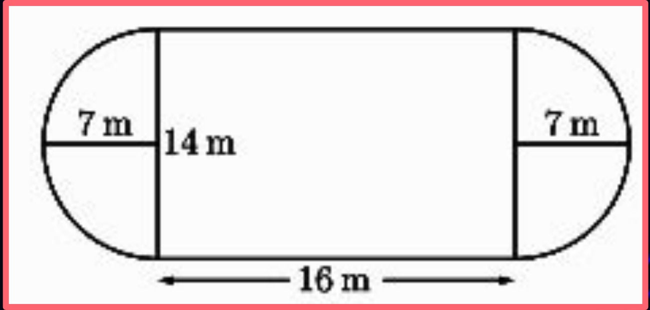
$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times 22 \times 3 \times 21$$

$$= 11 \times 21 = 231 \text{ cm}^2$$

Q. Find the area of the adjoining diagram.

[Board Term-2, 2014]



Solution

The given figure is combination of one rectangle and two semicircle of same radii .

Required area,

= area of two semi-circles + area of rectangle

= area of one circle + area of rectangle

$= \pi r^2 + (l \times b)$

(where r is radius of circle and l and b are length and breadth of rectangle)

$= \frac{22}{7} \times 7^2 + (16 \times 14)$

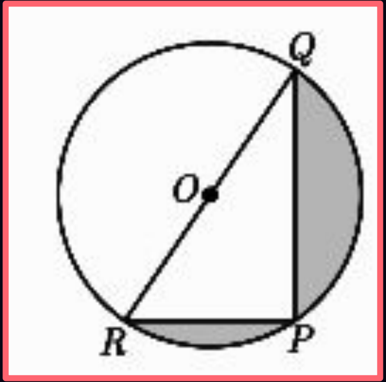
$= \frac{22}{7} \times 7 \times 7 + (16 \times 14)$

$= 154 + 224 = 378 \text{ m}^2$

Perimeter of shaded region is 31.4 cm approx.

Q. Find the area of the shaded region in Figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

[Board 2020 OD Standard]



Solution

$$
\begin{aligned}
\text{We have} \qquad PQ &= 24 \text{ cm} \\
PR &= 7 \text{ cm}
\end{aligned}
$$

The angle in the semicircle is right angle, therefore

$$
\begin{aligned}
\angle RPQ &= 90^\circ \\
\text{In } \Delta RPQ, \qquad RQ^2 &= PR^2 + PQ^2 \\
RQ^2 &= (7)^2 + (24)^2 \\
&= 49 + 576 = 625 \\
RQ &= 25 \text{ cm} \\
\text{Area of } \Delta RPQ &= \frac{1}{2} \times RP \times PQ \\
&= \frac{1}{2} \times 7 \times 24 \\
&= 84 \text{ cm}^2
\end{aligned}
$$

Solution

$$\text{area of semi-circle} = \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2$$

$$= \frac{11 \times 625}{7 \times 4} = \frac{6875}{28} \text{ cm}$$

Now, area of shaded region

$$= \text{area of semi-circle} - \text{area of } \Delta RPQ$$

$$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$$

$$= \frac{4523}{28}$$

$$= 161.54 \text{ cm}^2$$

Q. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm^2. If the wire is bent in the form of a circle, find the area enclosed by the circle. Use $\pi = 22/7$

[Board Term-2 OD 2017]

Solution

Let l be length of wire. If it is bent in the form of an equilateral triangle, side of triangle will be $\frac{l}{3}$.
Area enclosed by the triangle,

$$\frac{\sqrt{3}}{4} \times \left(\frac{l}{3}\right)^2 = 121\sqrt{3}$$

$$\frac{1}{4} \times \left(\frac{l}{3}\right)^2 = 121$$

$$\frac{1}{2} \times \frac{l}{3} = 11$$

$$l = 66 \text{ cm}$$

Same wire is bent in the form of circle. Thus circumference of circle will be 66.

$$2\pi r = 66$$

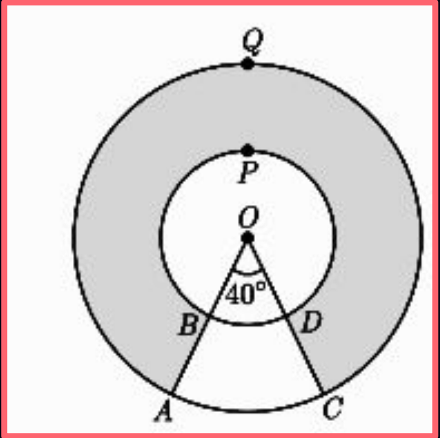
$$r = \frac{66}{2\pi} = \frac{66}{2 \times \frac{22}{7}} = \frac{21}{2}$$

Area enclosed by the circle

$$\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} = 346.5 \text{ cm}^2$$

Q. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40^0$. Use $\pi = 22/7$.

[Board Term-2 OD 2016]



Solution

Radii of two concentric circle is 7 cm and 14 cm.

Angle $\angle AOC = 40^\circ$,

Angle $\angle AOC = 360^\circ - 40^\circ = 320^\circ$

Area of shaded region,

$$\frac{\theta}{360^\circ}\pi[R^2 - r^2] = \frac{320^\circ}{360^\circ} \times \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{8}{9} \times 22 \times (14 \times 2 - 7)$$

$$= \frac{8}{9} \times 22 \times 21 = \frac{8}{3} \times 22 \times 7$$

$$= \frac{8 \times 154}{3} \text{ cm}^2$$

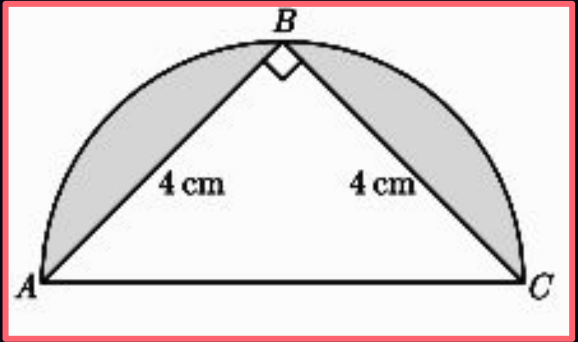
Required area,

$$= \frac{1232}{3} \text{ cm}^2$$

$$= 410.67 \text{ cm}^2$$

Q. In the figure, △ ABC is in the semi-circle, find the area of the shaded region given that AB = BC 4 cm.
(Use π = 3 .14)

[Board Term-2 Delhi 2014]



Solution

As ΔABC is a triangle in semi-circle, $\angle B$ is right angle,

$$AC = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ cm}$$

Radius of circle $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$ cm

Area of shaded portion,

$$= \text{Area of the semi-circle} - (\text{Area of } \Delta ABC)$$

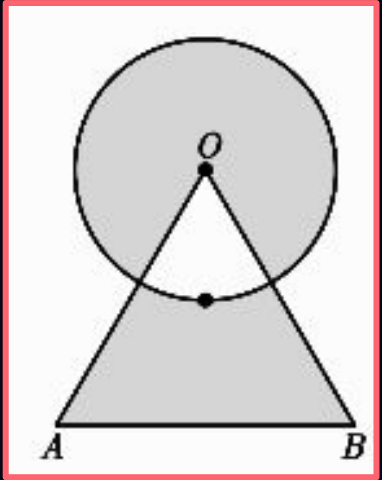
$$= \left\{\frac{1}{2}\pi \times (2\sqrt{2})^2\right\} - \left\{\frac{1}{2} \times 4 \times 4\right\}$$

$$= \left\{\frac{1}{2} \times 3.14 \times 8\right\} - 8$$

$$= 12.56 - 8 = 4.56 \text{ cm}^2$$

Q.Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

[Board Term-2 Foreign SQP 2016]]



Solution

Since OAB is an an equilateral triangle,we have

$$\angle AOB = 60^\circ$$

Area of shaded region = Area of major sector + (Area of $\Delta AOB-$ Area of minor sector)

$$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360} \times \frac{22}{7} \times 6^2\right)$$

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7} = 36\sqrt{3} + \frac{528}{7} \text{ cm}^2$$

Q. In fig. APB and AQP are semi-circle, and AO = OB . If the perimeter of the figure is 47 cm, find the area of the shaded region. Use π = 22/7

[Board Term-2 Delhi 2015]

Solution

Let r be the radius of given circle. It is given that perimeter of given figure is 47 cm.

$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$

$$\frac{3\pi r}{2} + 2r = 47$$

$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$

$$r\left(\frac{33}{7} + 2\right) = 47$$

$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

$$A = \text{area of circle} - \frac{1}{4} \text{ area of circle}$$

$$= \frac{3}{4} \text{ area of circle}$$

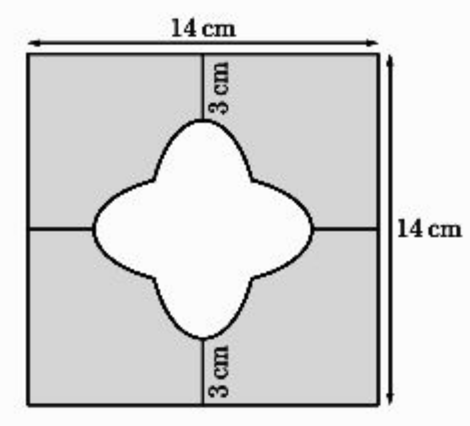
$$= \frac{3}{4}\pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{3}{2} \times 77$$

$$= 115.5 \text{ cm}^2$$

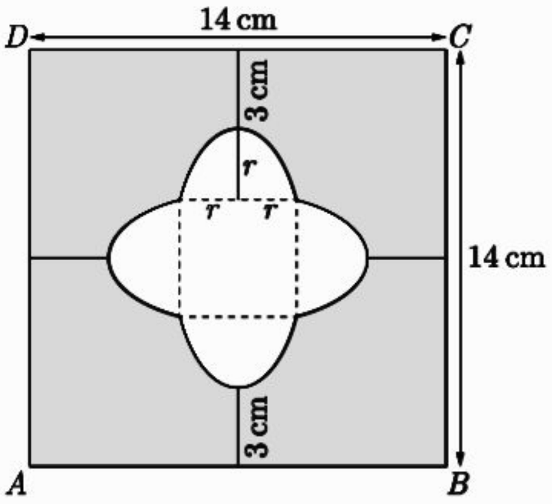
Q. In fig., find the area of the shaded region (π = 3.14)

[Board Term-2 2011, Delhi 2015]



Solution

We have redrawn the given figure as shown below.



$$3 + r + 2r + r + 3 = 14$$

$$4r + 6 = 14 \Rightarrow r = 2$$

Thus radius of the semi-circle formed inside is 2 cm and length of the side of square formed inside the semi-circle is 4 cm.

Area of square $ABCD$

$$= 14 \times 14 = 196 \text{ cm}^2$$

Solution

Thus area of 4 semi circle $= 4 \times \frac{1}{2}\pi r^2$

$$= 2 \times 3.14 \times 2 \times 2 = 25.12 \text{ cm}^2$$

Area of the square formed inside the semi-circle

$$(2r)^2 = 4 \times 4 = 16 \text{ cm}^2$$

Area of the shaded region,

$$= \text{area of square } ABCD$$

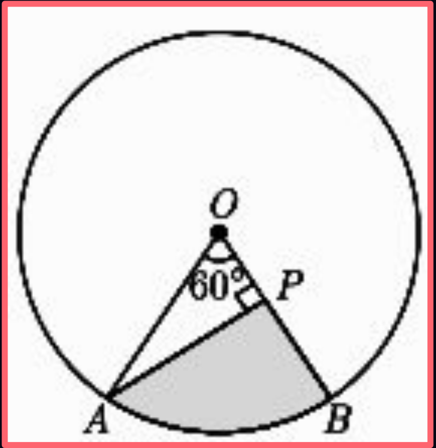
$$- (\text{Area of 4 semi-circle} + \text{Area of square})$$

$$= 196 - (25.12 + 16)$$

$$= 196 - 41.12 = 154.88 \text{ cm}^2$$

Q. In the given figure, AOB is a sector of angle 60^0 of a circle with centre O and radius 17 cm. If AP ⊥ OB and AP = 15 cm, find the area of the shaded region.

[Board Term-2 2016]



Solution

Here $OA = 17$ cm $AP = 15$ cm and ΔOPA is right triangle

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2 - 15^2} = 8 \text{ cm}$$

Area of the shaded region

$= $ Area of the sector $\Delta\, OAB$ – Area of ΔOPA

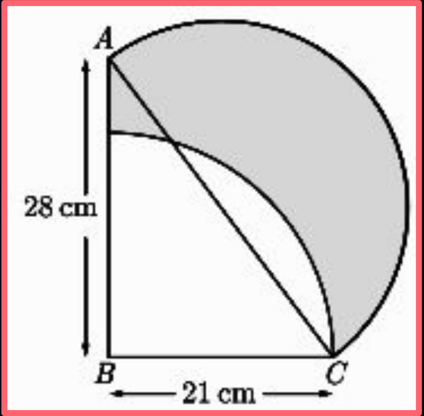
$$= \frac{60}{360} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$$

$$= 151.38 - 60 = 91.38 \text{ cm}^2$$

Q. In the fig., ABC is a right-angle triangle, $\angle B = 90^0$, AB = 28 cm and BC = 21 cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region

[Board Term -2 2011, Foreign 2014]



Solution

In right angled triangle ΔABC using Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$
$$= 28^2 + 21^2$$
$$= 784 + 441$$

or $$AC^2 = 1225 \Rightarrow AC = 35 \text{ cm}$$

Area of shaded region,

$= \text{area of } \Delta ABC +$

$+ \text{ area of semi-circle with diameter } AC +$

$- \text{ area of quadrant with radius } BC$

$$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2 - \frac{1}{4} \times \frac{22}{7} \times (21)^2$$

$$= 21 \times 14 + \frac{11}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$$

$$= 21 \times 14 + \frac{55}{2} \times \frac{35}{2} - \frac{11}{2} \times 3 \times 21$$

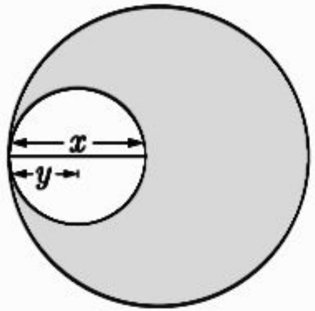
$$= 294 + 481.25 - 346.5$$
$$= 775.25 - 346.5 = 428.75 \text{ cm}^2.$$

Q. Two circles touch internally. The sum of their areas is 116 π and the difference between their centres is 6 cm. Find the radii of the circles.

[Board Term-2 Foreign 2017]

Solution

Let the radius of larger circle be x and the radius of smaller circle be y. As per question statement we have shown diagram below.



Now $$x - y = 6 \quad ...(1)$$

and $$\pi x^2 + \pi y^2 = 116\pi$$

$$\pi(x^2 + y^2) = 116\pi$$

$$x^2 + y^2 = 116 \quad ...(2)$$

Solution

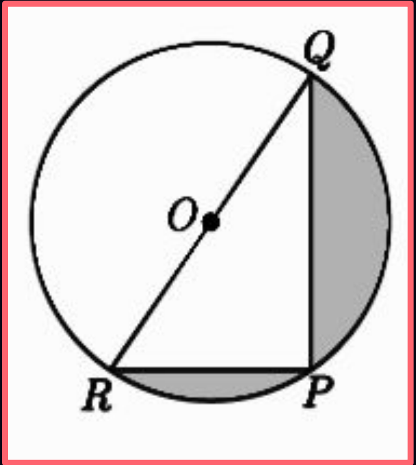
From (1) and (2) we have

$$\begin{aligned} x^2+(x-6)^2 &= 116 \\ x^2+x^2-12x+36 &= 116 \\ x^2-6x-40 &= 0 \\ x^2-10x+4x-40 &= 0 \\ x(x-10)+4(x+10) &= 0 \\ x &= 10, \text{ and } y = 10-6 = 4 \end{aligned}$$

Hence, radii of the circles are 10 cm and 4 cm.

Q. In the figure, O is the centre of circle such that diameter AB = 13 cm and AC = 12cm. BC is joined. Find the area of the shaded region. (π = 3 .14)

Board Term-2 OD 2016]



Solution

We redraw the given figure as below.
Radius of semi circle ACB,

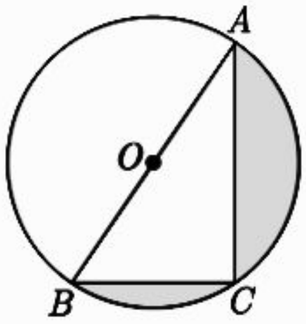
$$r = \frac{13}{2} \text{ cm}$$

Area of semicircle,

$$\frac{\pi}{2} r^2 = \frac{3.14}{2} \times \frac{13}{2} \times \frac{13}{2}$$

$$= \frac{3.14 \times 169}{8} = \frac{530.66}{8} \text{ cm}^2$$

The angle subtended on a semicircle is a right angle, thus $\angle ACB = 90^\circ$



Solution

In ΔABC,

$$AC^2 + BC^2 = AB^2$$

$$12^2 + BC^2 = 169$$

$$BC^2 = (160 - 144) = 25$$

$$BC = 5 \text{ cm}$$

Also area of triangle ΔABC,

$$\Delta = \frac{1}{2} \times \text{Base} \times \text{Hight}$$

$$= \frac{1}{2} \times AC \times BC$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

Area of shaded region,

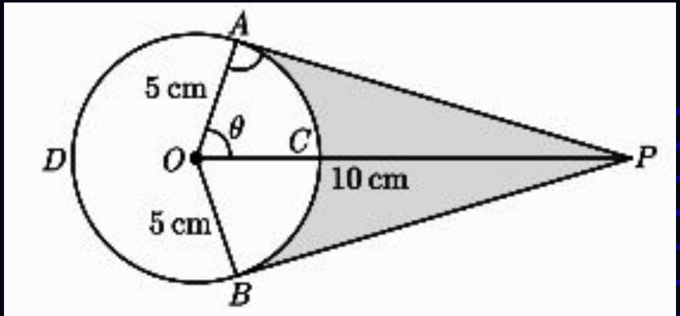
$$\frac{\pi}{2} r^2 - \Delta = \frac{530.66}{8} - 30$$

$$= (66.3325 - 30) \text{ cm}^2$$

$$= 36.3325 \text{ cm}^2$$

Q. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

[Board Term-2 Delhi 2016]



Solution

Here AP is tangent at point A on circle.

Thus $\angle OAP = 90^\circ$

Now $\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} = \cos 60^\circ$

Thus $\theta = 60^\circ$

Reflex $\angle AOB = 360^\circ - 60^\circ - 60^\circ = 240^\circ$

Now $\text{arc } ADB = \frac{2 \times 3.14 \times 5 \times 120^\circ}{360^\circ}$

$= 20.93 \text{ cm}$

Hence length of elastic in contact is 20.93 cm.

Now, $AP = 5\sqrt{3}$ dm

Area $(\Delta OAP + \Delta OBP) = 25\sqrt{3} = 43.25 \text{ cm}^2$

Area of sector $OACB$,

$= 25 \times 3.14 \times \frac{120^\circ}{360^\circ} = 26.16 \text{ cm}^2$.

Shaded Area $= 43.25 - 26.16 = 17.09 \text{ cm}^2$

Q. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

[Board Term-2 Foreign 2015]

Solution

Long hand makes 24 rounds in 24 hours and short hand makes 2 round in 24 hours. Distance travelled by tips of hands in one round is equal to the circumference of circle. Radius of the circle formed by long hand $= 6$ cm. and radius of the circle formed by short hand $= 4$ cm.

Distance travelled by long hand in one round

$$= \text{circumference of the circle } 2 \times 6 \times \pi$$

Distance travelled by long hand in 24 rounds

$$= 24 \times 12\pi = 288\pi$$

Distance travelled by short hand in a round $= 2 \times 4\pi$

Distance travelled by short hand in 2 round

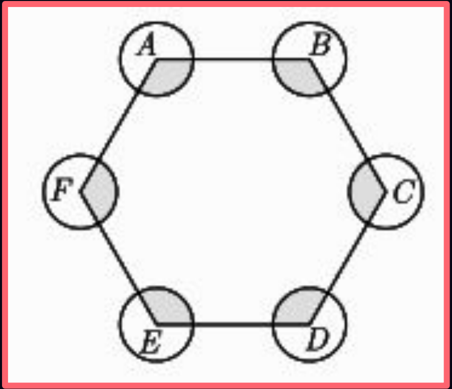
$$= 2 \times 8\pi = 16\pi$$

Sum of the distance $= 288\pi + 16\pi = 304\pi$

$$= 304 \times 3.14 = 954.56 \text{ cm}$$

Q. In fig., ABCDEF is any regular hexagon with different vertices A, B, C, D, E and F as the centres of circle with same radius *r* are drawn. Find the area of the shaded portion.

[Board Term-2 2011]



Solution

Let n be number of sides.

Now

$$n \times \text{each angle} = (n-2) \times 180^\circ$$

$$6 \times \text{each angle} = 4 \times 180^\circ$$

$$\text{each angle} = 120^\circ$$

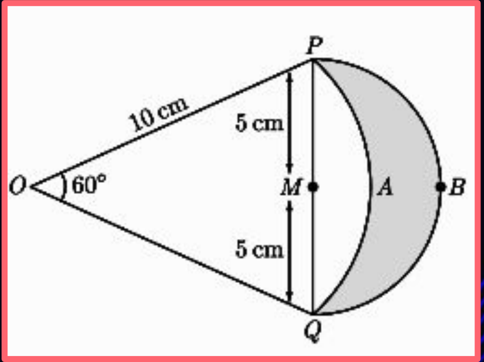
$$\text{Area of a sector} = \pi r^2 \times \frac{120^\circ}{360^\circ}$$

$$\text{Area of 6 shaded regions} = 6\pi r^2 \times \frac{120^\circ}{360^\circ}$$

$$= 2\pi r^2$$

Q. Figure shows two arcs PAQ and PQB . Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M . If OP = PQ = 10 cm show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)\text{cm}^2$

[Board Term-2 Delhi 2016]



Solution

We have $\angle POQ = 60^\circ$

and $OP = OQ = PQ = 10$

Area of segment $PAQM$,

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{ cm}^2$$

Area of semicircle $= \frac{\pi 5^2}{2} = \frac{25\pi}{2} \text{ cm}^2$

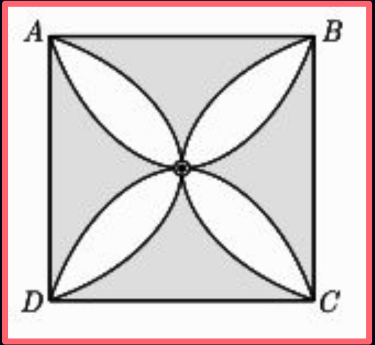
Area of shaded region,

$$= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2$$

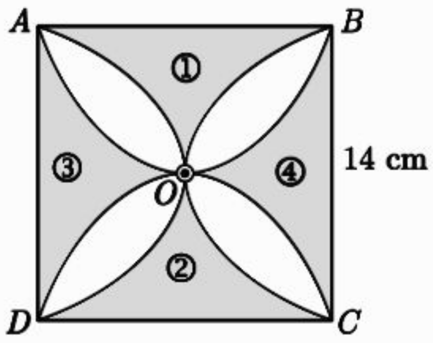
Q. In fig. ABCD is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = 22/7$

[Board Term-2 Delhi 2016, STD SQP 2021]



Solution

We have redrawn the given figure as shown below.



If we subtract area of two semicircle AOD and COB, from square $ABCD$ we will get area of part 1 and part 2.

Area of square $= 14 \times 14 = 196 \text{ cm}^2$

Radius of semicircle $= \frac{14}{2} = 7 \text{ cm}$

Area of semicircle $AOB + DOC$

$$= \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$$

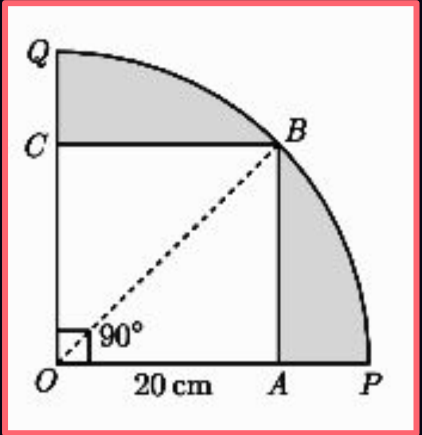
So, area of each of two shaded part

$$196 - 154 = 42 \text{ cm}^2$$

Hence, area of four shaded parts is 84 cm².

Q. A square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region. [Use $\pi = 3.14$]

[Board Term-2 Delhi 2014]



Solution

We have $OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{20^2 + 20^2} = \sqrt{800}$

Thus $OB = 20\sqrt{2}$ cm

Radius $r = 20\sqrt{2}$

Area of shaded region

$= \text{Area of sector } OQBPO - \text{ Area of square } OABC$

$$= \pi r^2 \frac{90^\circ}{360^\circ} - (20)^2$$

$$= 3.14 \times (20\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} - (20)^2$$

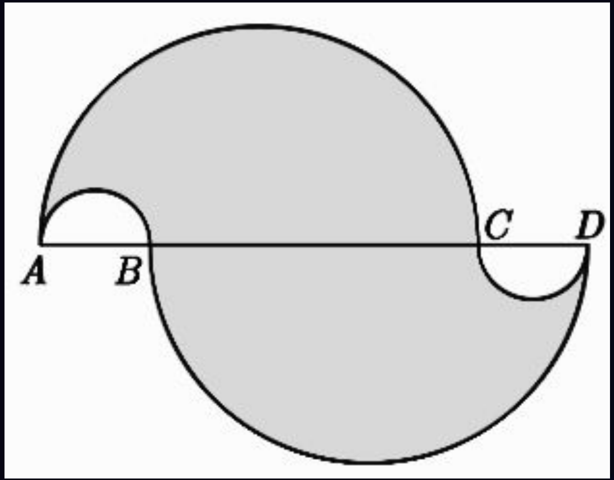
$$= 3.14 \times 200 - 400$$

$$= 628 - 400 = 228$$

Required area is 228 cm^2.

Q. In fig., AC = BD= 7 cm and AB = CD = 1. 75 cm. Semicircles are drawn as shown in the figure. Find the area of the shaded region. Use π = 22/7

[Board Term-2 2011]



Solution

Area of shaded region

$$= 2(\text{Area of semi-circle of radius } \tfrac{7}{2} \text{ cm}) - 2(\text{Area of semi-circle of radius } \tfrac{7}{8} \text{ cm})$$

$$= 2\left[\frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2\right] - 2\left[\frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{8}\right)^2\right]$$

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2\left[1 - \left(\frac{1}{4}\right)^2\right]$$

$$= \frac{77}{2}\left[1 - \frac{1}{16}\right] = \frac{77}{2} \times \frac{15}{16} = \frac{1155}{32} \text{ cm}^2$$

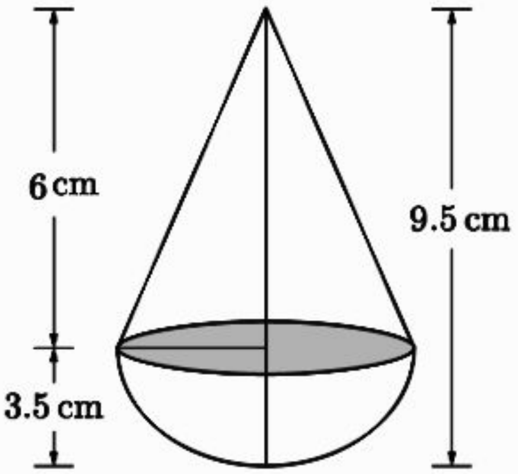
$$= 36.09 \text{ cm}^2$$

Q. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

[Board 2013]

Solution

As per question the figure is shown below. Here total volume of the toy is equal to the sum of volume of hemisphere and cone.



Solution

Volume of toy,

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times (6 + 2 \times 3.5)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times (6 + 7)$$

$$= \frac{1}{3} \times \frac{22}{2} \times 3.5 \times 13$$

$$= \frac{1}{3} \times 11 \times 3.5 \times 13$$

$$= \frac{500.5}{3} = 166.83 \text{ cm}^3 \qquad \text{(Approx)}$$

Hence, the volume of the solid is 166.83 cm^3.

Q. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

[Board 2011]

Solution

Here diameter of hemisphere is equal to the side of cubical block which is 7 cm.

Diameter of hemisphere = Side of cubical block

$$2r = 7 \Rightarrow r = \frac{7}{2}$$

Surface area of solid

$$= \text{Surface area of the cube}$$
$$- \text{Area of base of hemisphere}$$
$$+ \text{curved surface area of hemisphere}$$
$$= 6l^2 - \pi r^2 + 2\pi r^2$$
$$= 6l^2 + \pi r^2$$
$$= 6 \times 7^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$
$$= 6 \times 49 + \frac{77}{2} = 332.5 \text{ cm}^2$$

Q. Two cubes of 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.

[Board Term-2 2011]

Solution

Let l be the length of the cuboid so formed.

Now $l = 5 + 5 = 10$ cm, $b = 5$ cm; $h = 5$ cm.

$$\begin{aligned} \text{Surface area} &= 2(l \times b + b \times h + h \times l) \\ &= 2(10 \times 5 + 5 \times 5 + 5 \times 10) \\ &= 2(50 + 25 + 50) \\ &= 2 \times 125 = 250 \text{ cm}^2. \end{aligned}$$

Q. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3.

Find the height of the cylinder. Use π = 22/7

[Board 2020 OD Standard]

Solution

Let r be the radius of base of cylinder and h be height.

$$\text{Volume of a right circular cylinder} = 25\frac{1}{7} \text{ cm}$$

$$\pi r^2 h = \frac{176}{7}$$

$$\frac{22}{7} \times h^2 \times h = \frac{176}{7}$$

$$h^3 = \frac{176}{22} = 8 = 2^3.$$

Hence, height of the cylinder $= 2$ cm.

Q. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.

[Board Term-2 OD 2017]

Solution

Radius of cylinder and cone,

$$r = 0.7 \text{ cm}$$

Height of cylinder and cone,

$$h = 2.4 \text{ cm}$$

Slant height of cone,

$$l = \sqrt{r^2 + h^2} = \sqrt{0.7^2 + 2.4^2} = 2.5 \text{ m}$$

Total surface area of remaining solid,

$$= \text{CSA of cylinder} + \text{CSA of cone} + \text{Area of top.}$$

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 0.7(2 \times 2.4 + 2.5 + 0.7)$$

$$= \frac{22}{7} \times 0.7 \times 8 = \frac{176}{10}$$

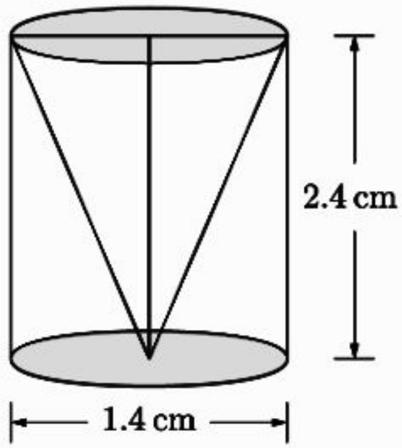
Hence total surface area is 17.6 cm^2

Q. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm3 .
Use π =22/7

[Board Term-2 2012]

Solution

As per question the figure is shown below.



Volume of remaining solid is difference of volume of cylinder and volume of cone.

$$\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times (0.7)^2 \times 2.4$$

$$= 44 \times 0.1 \times 0.7 \times 0.8$$

$$= 4.4 \times .56 = 2.464 \text{ cm}^3.$$

Q. If the total surface area of a solid hemisphere is 462 cm^2 , find its volume. Use Use π =22/7

[Board Term-2 Delhi 2014]

Solution

Total surface area of hemisphere,

$$3\pi r^2 = 462 \text{ cm}^2$$

$$\frac{22r^2}{7} = \frac{462}{3}$$

$$r^2 = \frac{462 \times 7}{22 \times 3} = 49$$

$$r = 7 \text{ cm.}$$

Volume of hemisphere,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{2156}{3} = 718.67 \text{ cm}^3.$$

Q. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

[Board Term-2 Foreign 2014, Delhi 2014]

Solution

We have radius $r = 7$ m and height $h = 24$ m

Slant height of tent,

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$
$$= \sqrt{625} = 25 \text{ m.}$$

Curved surface area of cone,

$$\pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Curves surface area of tent will be required area of cloth.
Let x meter of cloth is required

$$5x = 550 \text{ or, } x = \frac{550}{5} = 110 \text{ m.}$$

Thus 110 m of cloth is required.

$$\text{Cost of cloth} = 25 \times 110 = Rs.2750.$$

Q. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

[Board 2020 OD Standard]

Solution

Let h and r be the height and radius of cylinder and cone.

Height, $h = 14$ cm

and radius, $r = 6$ cm

Volume of the remaining solid,

$$V_{\text{remain}} = V_{\text{cylinder}} - V_{\text{cone}}$$

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^2$$

Q. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then find the volume of remaining solid?

[Board 2008]

Solution

Volume of the remaining solid

$$= \text{Volume of the cylinder} - \text{Volume of the cone}$$

$$= \pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10$$

$$= (360\pi - 120\pi) = 240\pi \text{ cm}^3$$

Q. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out, Find the total surface area of remaining solid. (Given your answer in terms of π).

[Board 2010]

Solution

Height of cylinder, $h = 15$ cm

Radius of cylinder, $r = \frac{16}{2} = 8$ cm

Radius of base of cone, $r = 8$ cm

Let slant height of cone be l, then we have

$$l = \sqrt{r^2 + h^2} = \sqrt{8^2 + 15^2}$$
$$= \sqrt{64 + 225} = \sqrt{289}$$

Thus $l = 17$ cm

TSA of reaming solid

= Top area of cylinder+

+ CSA of cylinder + CSA of conical cavity

$$= \pi r^2 + 2\pi rh + \pi rl$$
$$= \pi r(r + 2h + l)$$
$$= \pi \times 8(3 + 2 \times 15 + 17)$$
$$= \pi \times 8 \times 55 = 440\pi$$

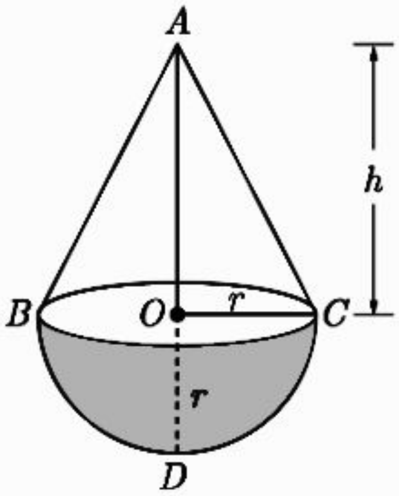
TSA of reaming solid is 440π.

Q. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

[Board 2020 OD Standard]

Solution

Let ABC be a cone, which is mounted on a hemisphere.



We have $OC = OD = r$

Curved surface area of the hemispherical part

$$= \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

Slant height of a cone, $l = \sqrt{r^2 + h^2}$

Solution

Slant height of a cone, $l = \sqrt{r^2 + h^2}$

Curved surface area of a cone $= \pi r l = \pi r\sqrt{h^2 + r^2}$

Since curved surface areas of the hemispherical part and the conical part are equal,

$$2\pi r^2 = \pi r\sqrt{h^2 + r^2}$$

$$2r = \sqrt{h^2 + r^2}$$

Squaring both of the sides, we have

$$4r^2 = h^2 + r^2$$

$$4r^2 - r^2 = h^2$$

$$3r^2 = h^2$$

$$\frac{r^2}{h^2} = \frac{1}{3}$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

Hence, the ratio of the radius and the height is $1 : \sqrt{3}$

Q. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

[Board 2020 OD Standard]

Solution

Let h and r be the height and radius of cylinder and cone.

Height, $h = 14$ cm

and radius, $r = 6$ cm

Volume of the remaining solid,

$$V_{remain} = V_{cylinder} - V_{cone}$$

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

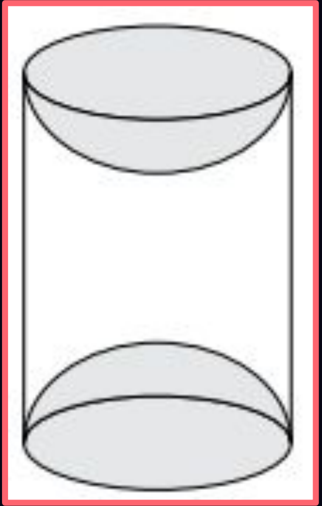
$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^3$$

Q. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.

[Board 2018]



Solution

Total surface Area of article

$= \text{CSA of cylinder} + \text{CSA of 2 hemispheres}$

$$\text{CSA of cylinder} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 10$$

$$= 220 \text{ cm}^2$$

Curved surface area of two hemispherical scoops

$$= 2 \times 2\pi r^2$$

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 154 \text{ cm}^2$$

Total surface area of article$= 220 + 154 = 374 \text{ cm}^2$

Q. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

[Board 2020 OD STD, 2019 Delhi, Delhi 2014, 2012]

Solution

Canal is the shape of cuboid where

$$\text{Breadth} = 6\text{ m}$$

$$\text{Depth} = 1.5\text{ m}$$

and $\quad \text{speed of water} = 10\text{ km/hr}$

Length of water moved in 60 minutes i.e. 1 hour

$$= 10\text{ km}$$

Length of water moved in 30 minutes i.e. $\frac{1}{2}$ hours,

$$= \frac{1}{2} \times 10 = 5\text{ km} = 5000\text{ m}$$

Now, volume of water moved from canal in 30 minutes

$$= \text{Length} \times \text{Breadth} \times \text{Depth}$$

$$= 5000 \times 6 \times 1.5\text{ m}^3$$

Solution

Volume of flowing water in canal

$$= \text{volume of water in area irrigated}$$

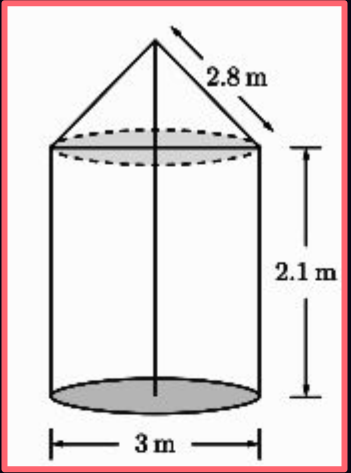
$$5000 \times 6 \times 1.5\text{m}^3 = \text{Area Irrigated} \times 8\,\text{cm}$$

$$5000 \times 6 \times 1.5\text{m}^3 = \text{Area Irrigated} \times \frac{8}{100}\,\text{m}$$

$$\text{Area Irrigated} = \frac{5000 \times 6 \times 1.5 \times 100}{8}\,\text{m}^2$$

$$= 5.625 \times 10^5\,\text{m}^2$$

Q. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = 22/7$



[Board Term-2 OD 2016]

Solution

Area of canvas required will be surface area of tent.

Height of cylinder $= 2.1$ m

Radius of cylinder $=$ radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent,

$$= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.}$$

$$= \pi rl + 2\pi rh = \pi r(l + 2h)$$

Thus $$\pi r(l + 2h) = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1)$$

$$= \frac{33}{7} \times 7 = 33 \text{ m}^2$$

$$\text{Total Cost} = 33 \times 500$$

$$= 16,500 Rs$$

Q. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\pi = 22/7$.

[Board Term-2 Delhi 2016]

Solution

We have $\qquad r + h = 37 \qquad (1)$

and $\qquad 2\pi r(r + h) = 1628 \qquad (2)$

Thus $\qquad 2\pi r \times 37 = 1628$

$$2\pi r = \frac{1628}{37} \Rightarrow r = 7 \text{ cm}$$

Substituting $r = 7$ in (1) we have

$$h = 30 \text{ cm.}$$

Here volume of cylinder

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

Q. A hemispherical bowl of internal diameter 36 cm contains liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

[Board Term-2 OD 2015]

Solution

Volume of the hemispherical bowl of internal diameter 36 cm will be equal to the 72 cylindrical bottles of diameter 6 cm.

$$\text{Volume of bowl} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$$

Volume of liquid in bowl is equal to the volume of bowl.

$$\text{Volume of liquid after wastage} = \frac{2}{3}\pi(18)^3 \times \frac{90}{100} \text{ cm}^3$$

$$\text{Volume of one bottle} = \pi r^2 h$$

Volume of liquid in 72 bottles

$$= \pi \times (3)^2 \times h \times 72 \text{ cm}^2$$

$$\text{Volume of bottles} = \text{volume in liquid after wastage}$$

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}$$

$$h = \frac{\frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

Hence, the height of bottle $= 5.4\text{cm}$

Q. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe

[Board Term-2, 2015]

Solution

Volume of copper used in making the pipe is equal to the difference of volume of external cylinder and volume of internal cylinder.

Height of cylindrical pipe,

$$h = 21 \text{ dm} = 210 \text{ cm}$$

External Radius, $R = \frac{10}{2} = 5 \text{ cm}$

Internal Radius, $r = \frac{6}{2} = 3 \text{ cm}$

Volume of copper used in making the pipe

$$= (\text{Volume of External Cylinder}) - (\text{Volume of Internal Cylinder})$$

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi h(R^2 - h^2)$$

Solution

$$= \frac{22}{7} \times 210 \times (5^2 - 3^2)$$

$$= \frac{22}{7} \times 210 \times (25 - 9)$$

$$= \frac{22}{7} \times 210 \times 16$$

$$= 10560 \text{ cm}^3.$$

Q. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left.
Use $\pi = 22/7$

[Board Term-2 OD 2014]

Solution

The diameter of the largest possible sphere is the side of the cube.

Side of cube $a = 7$ cm

Thus radius of sphere $r = \frac{7}{2}$ cm.

Volume of the wood left,

$$V_{cube} - V_{sphere} = a^3 - \frac{4}{3}\pi r^3$$

$$= 7^3 - \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$$

$$= 7^3\left[1 - \frac{4}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^3\right]$$

$$= 7^3\left[1 - \frac{4}{3} \times \frac{22}{7} \times \frac{1}{8}\right]$$

$$= 7^3\left[1 - \frac{11}{21}\right]$$

$$= 7^3 \times \frac{10}{21} = \frac{490}{3}$$

Hence, volume of wood $= 163.3$ cm^3.

Q. A well diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment.

[Board Term-2 Foreign 2017]

Solution

The volume of soil taken out from the well,

$$\pi^2 rh = \pi \times \left(\frac{3}{2}\right)^2 \times 14 \text{ m}^3$$

The radius of embankment with well

$$= \frac{3}{5} + 5 = \frac{13}{2} \text{ m}$$

Let the y be height of embankment. Then the volume of soil used in embankment,

$$\pi(R^2 - r^2)y = \pi r^2 h$$

$$\pi\left[\left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]y = \pi \times \left(\frac{3}{2}\right)^2 \times 14$$

$$\frac{160}{4}y = \frac{3}{2} \times \frac{3}{2} \times 14$$

$$y = \frac{3 \times 3 \times 14}{160} = 0.7875 \text{ m}$$

Hence the height of embankment is 78.75 cm.

Q. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

[Board Term-2 Delhi 2013]

Solution

Radius of pipe $r = \frac{2}{2} = 1$

Water flow rate $= 0.4 \text{ m/s} = 40 \text{ cm/s}$

Volume of water flowing through pipe in 1 sec.

$$\pi r^2 h = \pi \times (1)^2 \times 40 = 40\pi \text{ cm}^3$$

Volume of water flowing in 30 min $(30 \times 60 \text{ sec})$

$$= 40\pi \times 30 \times 60 = 72000\pi$$

Volume of water in cylindrical tank in 30 min,

Now $\pi R^2 H = \pi(40)^2 \times H$

$$\pi(40)^2 \times H = 72000\pi$$

$$40 \times 40 \times H = 72000\pi$$

Rise in water level

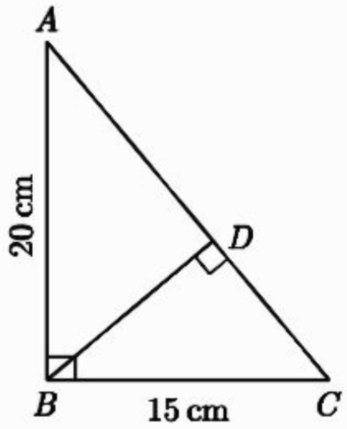
$$H = \frac{72000}{40 \times 40} = 45 \text{ cm.}$$

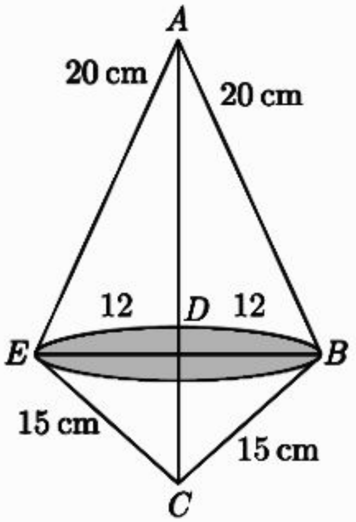
Thus level of water in the tank is 45 cm.

Q. A right triangle whose sides are 20 cm and 15 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use π = 3.1 4)

[Board Term-2 2012]

Solution





We have

$$AC^2 = 20^2 + 15^2 = 625$$

$$AC = 25 \text{ cm}$$

$$\text{area}(\Delta ABC) = \text{area}(\Delta ABC)$$

$$\frac{1}{2} \times AC \times BD = \frac{1}{2} \times BC \times AB$$

$$25 \times BD = 15 \times 20 = 300$$

$$BD = 12 \text{ cm}$$

Solution

Volume of double cone,

$$= \text{Volume of upper cone} + \text{Volume of lower cone}$$

$$= \frac{1}{3}\pi(BD)^2 \times AD + \frac{1}{3}\pi(BD)^2 \times CD$$

$$= \frac{1}{3}\pi(BD)^2(AD + CD)$$

$$= \frac{1}{3}\pi(BD)^2(AC)$$

$$= \frac{1}{3} \times 3.14 \times (12)^2 \times 25$$

$$= \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768 \text{ cm}^2$$

Surface area = CSA of upper cone + CSA of lower cone

$$= \pi(12)(20) + \pi(12)(15)$$

$$= 12\pi\{20 + 15\}$$

$$= 12 \times 3.14 \times 35$$

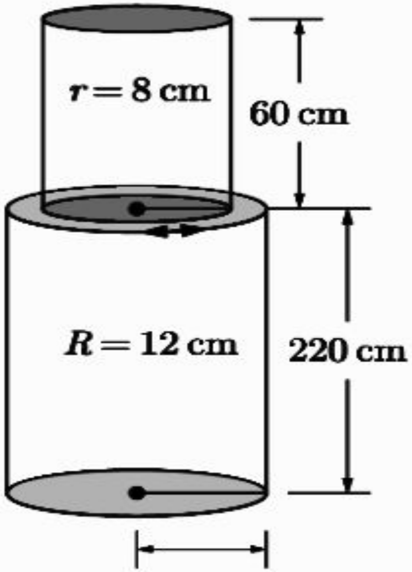
$$= 1318.8 \text{ cm}^2$$

Q. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pipe, given that 1 cm3 of iron has approximately 8 g mass. (Use π = 3.14)

[Board Term-2 Delhi 2014]

Solution

As per question the figure is shown below.



Radius of lower cylinder,	$R = 12$ cm
Height of lower cylinder,	$H = 220$ cm
Radius of upper cylinder,	$r = 8$ cm
Height of upper cylinder,	$h = 60$ cm

Solution

Volume of solid iron pole,

$$\pi R^2 H + \pi r^2 h = 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$$

$$= 111532.8 \text{ cm}^3$$

$$\text{Mass of pole} = 111532.8 \times 8 \text{ g}$$

$$= 892262.4 \text{ g}$$

$$= 892.2624 \text{ kg.}$$

Q. A heap of wheat is in the form of cone of diameter 6 m and height 3.5 m. Find its volume . How much canvas cloth is required to just cover the heap ? Use π = 22/7

[Board Term-2 Delhi 2016]

Solution

Radius of cone, $r = \frac{6}{2} = 3$ cm

Height of cone, $h = 3.5$ cm

Volume of wheat in the form of cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 3.5$$

$$= 11 \times 3 = 33 \text{ m}^3$$

$$l = \sqrt{3^2 + 3.5} = 4.609 \text{ m}$$

Canvas required to cover the heap,

$$\pi r l = \frac{22}{7} \times 3 \times 4.609$$

$$= 43.45 \text{ m}^2.$$

Q. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

[Board Term-2 OD 2014]

Solution

Radius of spherical marble $r_1 = \frac{1.4}{2} = 0.7$ cm

Radius of cylindrical vessel $R = \frac{7}{2} = 3.5$ cm

Let h be the rise in water level then,

Volume of 150 spherical marbles = Volume of water rise

$$150 \times \frac{4\pi}{3} \times \left(\frac{7}{10}\right)^3 = \pi \times \left(\frac{7}{2}\right)^2 \times h$$

$$150 \times \frac{4}{3} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{7}{2} \times \frac{7}{2} \times h$$

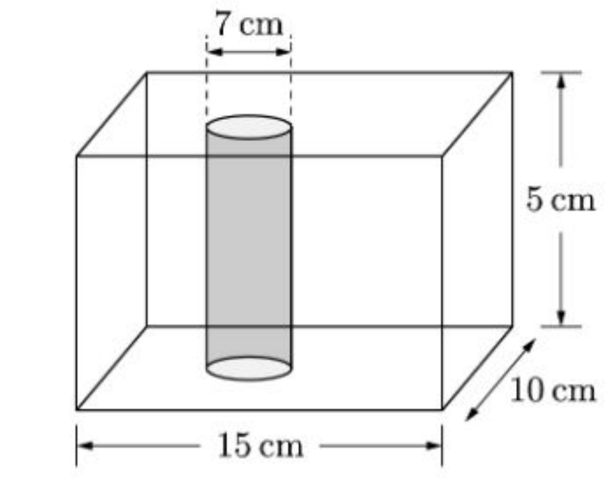
$$h = \frac{4 \times 7}{5}$$

$$\frac{28}{5} = h \Rightarrow h = 5.6 \text{ cm}$$

Thus 5.6 cm will be rise in the level of water.

Q. In fig., from a cuboidal solid metallic block of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. Use π = 22/7.

[Board Term-2 Delhi 2015]



Solution

We have $l = 15$ cm, $b = 10$ cm, $h = 5$cm, $r = \frac{7}{2}$ cm

$$\text{Total Surface area} = 2(lb + bh + hl) + 2\pi rh - 2\pi r^2$$

TSA of cuboidal block

$$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$$

$$= 550 \text{ cm}^2.$$

Area of curved surface cylinder,

$$2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 = 110 \text{ cm}^2$$

$$\text{Area of two circular bases} = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 77 \text{ cm}^2$$

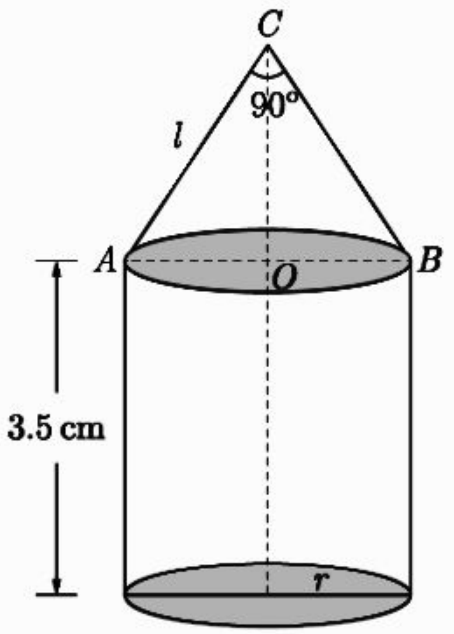
$$\text{Required area} = 550 + 110 - 77 = 583 \text{ cm}^3.$$

Q. A toy is in the form of a cylinder of diameter $2\sqrt{2}$ m and height 3.5 m surmounted by a cone whose vertical angle is 90^0. Find total surface area of the toy.

[Board Term-2 2012]

Solution

As per question the figure is shown below.



Here $\angle C = 90^\circ$ and $AC = BC = l$

Thus $\quad AB^2 = AC^2 + BC^2$

$= l^2 + l^2 = 2l^2$

Solution

Now $(2\sqrt{2})^2 = 2l^2$

Thus $l = 2$ and $r = \sqrt{2}$ m

Slant height of conical portion, $l = 2$ m

Total surface area of toy

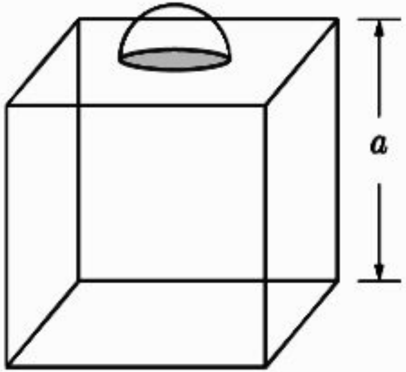
$$\begin{aligned} 2\pi rh + \pi r^2 + \pi rl &= \pi r[7 + \sqrt{2} + 2] \text{ m}^2 \\ &= \pi\sqrt{2}[9 + \sqrt{2}] \text{ m}^2 \\ &= \pi[2 + 9\sqrt{2}] \text{ m}^2 \end{aligned}$$

Q. A decorative block, made up of two solids - a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. Use $\pi = 22/7$.

[Board Term-2 Delhi 2016]

Solution

Let a be the side of cube and r be the radius of hemisphere. As per question the figure is shown below.



Surface area of block

$$= 6a^2 - \pi r^2 + 2\pi r^2$$

$$= 6a^2 + \pi r^2$$

$$= 6 \times (6)^2 + \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

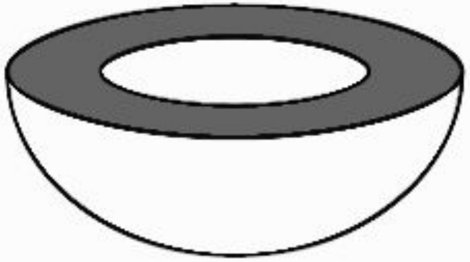
$$= 225.625 \text{ cm}^2.$$

Q. The internal and external diameters of a hollow hemispherical vessel are 16 cm and 12 cm respectively. If the cost of painting 1 cm^2 of the surface area is
Rs. 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

[Board Term-2 Delhi 2015]

Solution

As per question the figure is shown below.



$$\text{Here} \quad R = 8 \text{ cm}, \ r = 6 \text{ cm}$$

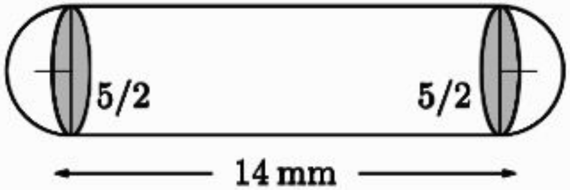
$$\begin{aligned}
\text{Surface area} &= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) \\
&= \pi[2 \times 8^2 + 2 \times 6^2 + (8^2 - 6^2)] \\
&= \pi[2 \times 64 + 2 \times 36 + (64 - 36)] \\
&= \pi[128 + 72 + 28] \\
&= 228 \times 3.14 = 715.92 \text{ cm}^2 \\
\text{Total cost} &= 715.92 \times 5 = 3579.60 \text{ Rs}
\end{aligned}$$

Q. A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 15 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule

[Board Term-2 2012]

Solution

As per question the figure is shown below.



Total height $= 14$ mm

Height of cylinder $= 14 - 2 \times 2.5 = 9$ mm

Radius of cylinder $= 2.5$ mm

Radius of hemisphere $= 2.5$ mm

Volume of capsule = Volume of two hemispheres + Volume of cylinder

Solution

$$= 2 \times \frac{2\pi r^3}{3} + \pi r^2 h$$

$$= \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 + \pi\left(\frac{5}{2}\right)^2 \times 9$$

$$= \pi\left(\frac{5}{2}\right)^2\left(\frac{4}{3} \times \frac{5}{2} + 9\right)$$

$$= \frac{25\pi}{4}\left(\frac{10}{3} + 9\right)$$

$$= \frac{25}{4}\pi\left(\frac{10 + 27}{3}\right) = \frac{25}{4}\pi\left[\frac{37}{3}\right]$$

$$= \frac{25}{4} \times \frac{22}{7} \times \frac{37}{3} = \frac{10175}{42} \text{ mm}^3$$

$$= 242.26 \text{ mm}^3.$$

STATISTICS

Q. If the mean of the following data is 14.7, find the values of p and q .

[Board 2013]

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42	Total
Frequency	10	p	4	7	q	4	1	40

Solution

Class	x_i	f_i	$f_i x_i$
0-6	3	10	30
6-12	9	p	$9p$
12-18	15	4	60
18-24	21	7	147
24-30	27	q	$27q$
30-36	33	4	132
36-42	39	1	39
	Total	$\sum f_i = 26 + p + q = 40$	$\sum f_i x_i = 408 + 9p + 27q$

We have

$$\begin{aligned} \sum f_i &= 40, \\ 26 + p + q &= 40 \\ p + q &= 14 \end{aligned} \quad \ldots(1)$$

Solution

Mean $M = \frac{\sum x_i f_i}{\sum f_i}$

$$14.7 = \frac{408 + 9p + 27q}{40}$$

$$588 = 408 + 9p + 27q$$

$$180 = 9p + 27q$$

$$p + 3q = 20 \qquad \ldots(2)$$

Subtracting equation (1) from (2) we have,

$$2q = 6 \Rightarrow q = 3$$

Substituting this value of q in equation (2) we get

$$p = 14 - q = 14 - 3 = 11$$

Hence, $p = 11,\ q = 3$

Q. Compute the median from the following data :

[Board 2015]

Mid-values	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

Solution

Mid-value	Class Groups	Frequency	Cumulative Frequency
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	106-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390

Solution

Now $N = 390\,;\ \frac{N}{2} = 195$

Cumulative frequency just greater than $\frac{N}{2}$ is 36 and the corresponding class is 150-160. Thus median class is 150-160.

Here, $l = 150,\ f = 116,\ h = 10,\ F = 151$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$= 150 + \frac{195 - 151}{116} \times 10$$

$$= 153.8$$

Q. Find the mode of the following frequency distribution

[Board 2019]

Class Interval	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Solution

Class Interval	Frequency
25-30	25
30-35	34
35-40	50
40-45	42
45-50	38
50-55	14

Class 35-40 has the maximum frequency 50, therefore this is model class.

Now, $l = 35$ $f_1 = 50$, $f_0 = 34$, $f_2 = 42$, $h = 5$

Solution

Now, $l = 35$ $f_1 = 50$, $f_0 = 34$, $f_2 = 42$, $h = 5$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 35 + \left(\frac{50 - 34}{2 \times 50 - 34 - 42}\right) \times 5$$

$$= 35 + \frac{16 \times 5}{24}$$

$$= 35 + \frac{10}{3} = \frac{105 + 10}{3} = \frac{115}{3} = 38.33$$

Q. Find the mean of the data using empirical formula when it is given that mode is 50.5 and median in 45.5.

[Board 2019]

Solution

Mode, $M_o = 50.5$

Median, $M_d = 45.5$

Now $3M_d = M_0 + 2M$

$$3 \times 45.5 = 50.5 + 2M$$

Mean $M = \frac{136.5 - 50.5}{2} = 43$

Hence mean is 43.

Q: The mean of the following frequency distribution is 25. Find the value of p.

CBSE 2015

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

Solution

Class-Interval	Mid-Point x_i	f_i	$f_i x_i$
0-10	5	4	20
10-20	15	6	90
20-30	25	10	250
30-40	35	6	210
40-50	45	p	$45\ p$
		$26 + p$	$570 + 45p$

We have
$$M = \frac{\sum f_i x_i}{\sum f_i}$$

$$25 = \frac{570 + 45p}{26 + p}$$

$$650 + 25p = 570 + 45p$$

$$650 - 570 = 45p - 25p$$

Thus
$$p = 4$$

Q: Find the mode of the following distribution of marks obtained by the students in an examination :

CBSE 2017

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	15	18	21	29	17

Solution

Class 60-80 has the maximum frequency 29, therefore this is model class.

Here, $l = 60$, $f_1 = 29$, $f_0 = 21$, $f_2 = 17$ and $h = 20$

$$\text{Mode,} \quad M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 60 + \frac{8}{58 - 38} \times 20$$

$$= 60 + 8 = 68$$

$$\text{Now} \quad 3M_d = M_o + 2M$$

$$= 68 + 2 \times 53$$

$$M_d = \frac{174}{3} = 58$$

Hence median is 58.

Q:The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

CBSE 2020

Class	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Solution

We prepare cumulative frequency table as given below.

Class Interval	Frequency (f)	Cumulative frequency $c.f.$
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
	$N=100$	

Solution

From table we have

$$76 + x + y = 100$$

$$x + y = 100 - 76 = 24 \qquad \ldots(1)$$

Here median is 525 which lies between class 500 – 600.
Thus median class is 500-600.

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$$25 = (50 - 36 - x)5$$

$$14 - x = \frac{25}{5} = 5$$

$$x = 14 - 5 = 9$$

Substituting the value of x is equation (1), we get

$$y = 24 - 9 = 15$$

Hence, $x = 9$ and $y = 15$

PROBABILITY

Q. A number is chosen at random from the numbers -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5. Then the probability that square of this number is less than or equal to 1 is

[Board 2020 SQP]

Solution

Given numbers are $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ and their squares are 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25.
Total number of outcomes $n(S) = 11$.
Favourable outcome are -1, 0, 1, thus number of favourable outcomes is $n(E) = 3$.

Required probability, $$P(E) = \frac{n(E)}{n(S)} = \frac{3}{11}$$

Q. There are 1000 sealed envelopes in a box. 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well-shuffled and an envelope is picked up out, then the probability that is contains no cash prize is

Solution

Total number of envelopes in the box $= 1000$

Number of envelopes containing cash prize

$$= 10 + 100 + 200 = 310$$

Number of envelopes containing no cash

$$= 1000 - 310 = 690$$

Now $\quad n(S) = 1000$

$$n(E) = 690$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{690}{1000} = 0.69$$

Q. A bag contains 18 balls of which x balls are red.

(i) If one ball is drawn at random from the bag, what is the probability that it is not red?

(ii) If 2 more red balls are put in the bag, the probability of drawing a red ball will be 9/8 times the probability of drawing a red ball in the first case. Find the value of x.

[Board 2015]

Solution

Total ball, $n(S) = 18$

Red ball $n(R) = x$

(i) not red

$P(\text{red ball}),\quad P(R) = \frac{n(R)}{n(S)} = \frac{x}{18}$

$$P\left(\bar{R}\right) = 1 - \frac{x}{18} = \frac{18 - x}{18}$$

(ii) Now two more red balls are added.

Now total ball $n'(S) = 18 + 2 = 20$

There ar total $x + 2$ red ball.

$$n'(R) = x + 2$$

$P(\text{red balls}),\quad P'(R) = \frac{n'(R)}{n'(S)} = \frac{x+2}{20}$

Solution

Now, according to the question,

$$\frac{x+2}{20}=\frac{9}{8}\times\frac{x}{18}$$

$$180x=144x+288$$

$$36x=288$$

$$x=\frac{288}{36}=8$$

Now substituting $x=8$ we have

$$P\left(\bar{R}\right)=\frac{18-8}{18}=\frac{10}{18}=\frac{5}{9}$$

Q. All the black face cards are removed from a pack of 52 cards. Find the probability of getting a

(i) face card

(ii) red card

(iii) black card

(iv) king

[Board 2014]

Solution

There are $52 - 6 = 46$ cards after removing black face cards. We have 46 cards and thus there are 48 possible outcomes.

$$n(S) = 46$$

(i) face card

Number of red cards, $n(E_1) = 12 - 6 = 6$

P(face card), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{46} = \frac{3}{23}$

(ii) red card

Number of red card, $n(E_2) = 26$

P(red card), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{26}{46} = \frac{13}{23}$

(iii) black card

Number of black card, $n(E_3) = 26 - 6 = 20$

P(black card), $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{20}{46} = \frac{10}{23}$

(iv)king

Number of king, $n(E_4) = 4 - 2 = 2$

P(king), $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{2}{46} = \frac{1}{23}$

Q. A die is thrown twice. Find the probability that

(i) 5 will come up at least once.

(ii) 5 will not come up either time.

[Board 2019]

Solution

There are $6 \times 6 = 36$ possible outcome.

Thus sample space for two die is

$$n(S) = 36$$

(i) 5 will come up at least once Favourable case are $(5,1)$, $(5,2)$, $(5,3)$, $(5,4)$, $(5,5)$, $(5,6)$, $(1,5)$, $(2,5)$, $(3,5)$, $(4,5)$ and $(6,5)$ thus 11 case.

Number of favourable outcome,

$$n(E_1) = 1$$

Probability that 5 will come up at least once,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{11}{36}$$

(ii) 5 will not come up either time

Probability that 5 will come up either time

$$P\left(\bar{E}\right) = 1 - P(E)$$

$$= 1 - \frac{11}{36} = \frac{36 - 11}{36} = \frac{25}{36}$$

MCQ
and
Assertion/Reason Questions

Q. The product of a non-zero rational and an irrational number is

[Board 2022 Term 1 SQP STD]

a. always irrational

b. always rational

c. rational or irrational

d. one

Solution

Product of a non-zero rational and an irrational number is always irrational i.e., $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ which is irrational. Thus (a) is correct option.

Q. The sum of exponents of prime factors in the prime factorisation of 196 is

[Board 2020 OD Standard]

a) 3

b) 4

c) 5

d) 2

Solution

Prime factors of 196,

$$\begin{aligned} 196 &= 4 \times 49 \\ &= 2^2 \times 7^2 \end{aligned}$$

The sum of exponents of prime factor is $2 + 2 = 4$.

Thus (b) is correct option.

Q. The total number of factors of prime number is

[Board 2020 Delhi Standard]

a 1

b 0

c 2

d 3

Solution

There are only two factors (1 and number itself) of any prime number.

Thus (c) is correct option.

Q. The HCF and the LCM of 12, 21, 15 respectively are

[Board 2020]

a. 3, 140

b. 12, 420

c. 3, 420

d. 420, 3

Solution

We have

$$\begin{aligned} 12 &= 2 \times 2 \times 3 \\ 21 &= 3 \times 7 \\ 15 &= 3 \times 5 \\ \text{HCF}(12, 21, 15) &= 3 \\ \text{LCM}\ (12, 21, 15) &= 2 \times 2 \times 3 \times 5 \times 7 = 420 \end{aligned}$$

Thus (c) is correct option.

Q. The LCM of smallest two digit composite number and smallest composite number is

[Board 2020]

a) 12

b) 4

c) 20

d) 44

Solution

Smallest two digit composite number is 10 and smallest composite number is 4.

$$\text{LCM } (10, 4) = 20$$

Thus (c) is correct option.

Q. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x , y are prime numbers, then HCF (a, b) is

a) xy

b) xy^2

c) x^3y^3

d) x^2y^2

Solution

We have

$$a = x^3 y^2 = x \times x \times x \times y \times y$$

$$b = xy^3 = x \times y \times y \times y$$

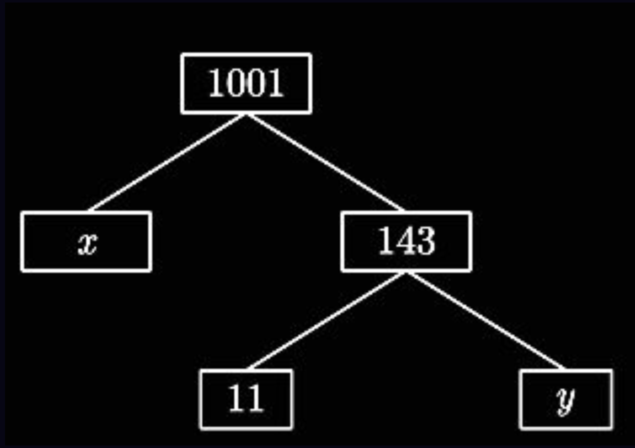
$$\text{HCF}(a, b) = \text{HCF}(x^3 y^3, xy^3)$$

$$= x \times y \times y = xy^2$$

HCF is the product of the smallest power of each common prime factor involved in the numbers.

Thus (b) is correct option.

Q. The values of x and y in the given figure are



a) 7, 13

b) 13, 7

c) 9, 12

d) 12, 9

Solution

$$1001 = x \times 143 \Rightarrow x = 7$$

$$143 = y \times 11 \Rightarrow y = 13$$

Hence $x = 7,\ y = 13$

Thus (a) is correct option.

Q. The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

a 10

b 100

c 504

d 2520

Solution

Factor of 1 to 10 numbers

$$\begin{aligned}
1 &= 1 \\
2 &= 1 \times 2 \\
3 &= 1 \times 3 \\
4 &= 1 \times 2 \times 2 \\
5 &= 1 \times 5 \\
6 &= 1 \times 2 \times 3 \\
7 &= 1 \times 7 \\
8 &= 1 \times 2 \times 2 \times 2 \\
9 &= 1 \times 3 \times 3 \\
10 &= 1 \times 2 \times 5 \\
\text{LCM}(1 \text{ to } 10) &= \text{LCM}\,(1,2,3,4,5,6,7,8,9,10) \\
&= 1 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \\
&= 2520
\end{aligned}$$

Thus (d) is correct option.

Q.
1. The L.C.M. of x and 18 is 36.
2. The H.C.F. of x and 18 is 2.
What is the number x ?

a 3

b 4

c 1

d 2

Solution

$$\text{LCM} \times \text{HCF} = \text{First number} \times \text{second number}$$

$$\text{Hence,} \qquad \text{required number} = \frac{36 \times 2}{18} = 4$$

Thus (d) is correct option.

Q. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

a) 240

b) 1600

c) 2400

d) 3600

Solution

The LCM of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect square number. 1600 is not multiple of 240.
Thus (d) is correct option.

Q. Assertion : The H.C.F. of two numbers is 16 and their product is 3072.
Then their L.C.M. = 162.
Reason : If a b, are two positive integers, then HCF ❌ LCM = a ❌ b .

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

Ans : (d) Assertion (A) is false but reason (R) is true.

Here reason is true [standard result]

Assertion is false.

$$\frac{3072}{16} = 192 \neq 162$$

Q. Assertion : The HCF of two numbers is 5 and their product is 150, then their LCM is 30
Reason : For any two positive integers a and b, HCF(a, b) + LCM(a, b) = a × b.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We have,

$$\begin{aligned} \mathrm{LCM}(a, b) \times \mathrm{HCF}(a, b) &= a \times b \\ \mathrm{LCM} \times 5 &= 150 \\ \mathrm{LCM} &= \frac{150}{5} = 30 \end{aligned}$$

Thus (c) is correct option.

Q. Assertion : 2 is a rational number.

Reason : The square roots of all positive integers are irrationals.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

Assertion (A) is true but reason (R) is false.
Here reason is not true, $\sqrt{4} = \pm 2$, which is not an irrational number.

Q. If α and β are zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is

[Board 2022 Term 1 2022 STD]

a) $\frac{15}{4}$

b) $\frac{-15}{4}$

c) 4

d) 15

Solution

We have $f(x) = x^2 - x - 4$

$$\alpha + \beta = -\frac{-1}{1} = 1 \text{ and } \alpha\beta = \frac{-4}{1} - 4$$

Now $$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta$$

$$= -\frac{1}{4} + 4 = \frac{15}{4}$$

Thus (a) is correct option.

Q. If the square of difference of the zeroes of the quadratic polynomial $x^2 + px + 45$ is equal to 144, then the value of p is

[Board 2022 Term 1 2022]

a. ± 9

b. ± 12

c. ± 15

d. ± 18

Solution

We have $f(x) = x^2 + px + 45$

Then, $\alpha + \beta = \frac{-p}{1} = -p$

and $\alpha\beta = \frac{45}{1} = 45$

According to given condition, we have

$$(\alpha - \beta)^2 = 144$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$(-p)^2 - 4(45) = 144$$

$$p^2 = 144 + 180 = 324 \Rightarrow p = \pm 18$$

Q. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

[Board 2020 Delhi Standard]

a. 10

b. −10

c. −7

d. −2

Solution

We have $p(x) = x^2 + 3x + k$

If 2 is a zero of $p(x)$, then we have

$$\begin{aligned} p(2) &= 0 \\ (2)^2 + 3(2) + k &= 0 \\ 4 + 6 + k &= 0 \\ 10 + k &= 0 \Rightarrow k = -10 \end{aligned}$$

Thus (b) is correct option.

Q. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

[Board 2020 Delhi Standard]

a) $x^2 + 5x + 6$

b) $x^2 - 5x + 6$

c) $x^2 - 5x - 6$

d) $-x^2 + 5x + 6$

Solution

Let α and β be the zeroes of the quadratic polynomial, then we have

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

Now
$$\begin{aligned} p(x) &= x^2 - (\alpha + \beta)x + \alpha\beta \\ &= x^2 - (-5)x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$

Thus (a) is correct option.

Q. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

[Board 2020]

a. $m, m + 3$

b. $-m, m + 3$

c. $m, -(m + 3)$

d. $-m, -(m + 3)$

Solution

We have $p(x) = x^2 - 3x - m(m+3)$

Substituting $x = -m$ in $p(x)$ we have

$$p(-m) = (-m)^2 - 3(-m) - m(m+3)$$
$$= m^2 + 3m - m^2 - 3m = 0$$

Thus $x = -m$ is a zero of given polynomial.
Now substituting $x = m+3$ in given polynomial we have

$$p(x) = (m+3)^2 - 3(m+3) - m(m+3)$$
$$= (m+3)[m+3-3-m]$$
$$= (m+3)[0] = 0$$

Thus $x = m+3$ is also a zero of given polynomial.
Hence, $-m$ and $m+3$ are the zeroes of given polynomial.
Thus (b) is correct option.

Q. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

[Board 2020 OD Standard]

a) both positive

b) both negative

c) one positive and one negative

d) both equal

Solution

Let $f(x) = x^2 + 99x + 127$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = 99$ and $c = 127$.

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = -99$

Product of zeroes $\alpha\beta = \frac{c}{a} = 127$

Now, product is positive and the sum is negative, so both of the numbers must be negative.

Q. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and −3, then

a) $a = -7, b = -1$

b) $a = 5, b = -1$

c) $a = 2, b = -6$

d) $a = 0, b = -6$

Solution

If a is zero of the polynomial, then $f(a) = 0$.
Here, 2 and -3 are zeroes of the polynomial $x^2 + (a+1)x + b$

So, $$f(2) = (2)^2 + (a+1)(-3) + b = 0$$

$$4 + 2a + 2 + b = 0$$

$$6 + 2a + b = 0$$

$$2a + b = -6 \quad \ldots(1)$$

Again, $$f(-3) = (-3)^2 + (a+1)2 + b = 0$$

$$9 - 3(a+1) + b = 0$$

$$9 - 3a - 3 + b = 0$$

$$6 - 3a + b = 0$$

$$-3a + b = -6$$

$$3a - b = 6 \quad \ldots(2)$$

Adding equations (1) and (2), we get

$$5a = 0 \Rightarrow a = 0$$

Substituting value of a in equation (1), we get

$$b = -6$$

Hence, $a = 0$ and $b = -6$.
Thus (d) is correct option.

Q. The value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously, is

a) 2

b) −2

c) −1

d) 1

Solution

Both expression $(x-1)(x+1)$ and $(x-1)(x-1)$ have 1 as zero. This both vanish if $x=1$.

Thus (d) is correct option.

Q. Assertion : If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$.

Reason : If $(x - \alpha)$ is a factor of $p(x)$, then $p(\alpha) = 0$ i.e. α is a zero of $p(x)$.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$$

$$1 = \frac{4k}{k^2+4}$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0 \Rightarrow k = 2$$

Assertion is true Since, Reason is not correct for Assertion.
Thus (b) is correct option.

Q. Assertion : If both zeros of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.
Reason : Sum of zeros of a quadratic polynomial $ax^2 + bx + c$ is $-b/a$

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

As the polynomial is $x^2 - 2kx + 2$ and its zeros are equal but opposition sign, sum of zeroes must be zero.

$$\text{sum of zeros} = 0$$

$$\frac{-(-2k)}{1} = 0 \Rightarrow k = 0$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Top 10

Most Important Questions

Q. Two lines are given to be parallel. The equation of one of the lines is $3x - 2y = 5$. The equation of the second line can be

(A) $9x + 8y = 7$

(B) $-12x8y = 7$

(C) $-12x + 8y = 7$

(D) $12x + 8y = 7$ [CBSE Board Term-I, 2021]

SOLUTION

1. **Option (C) is correct.**

Explanation: The given equation is

$$3x - 2y = 5$$

According to the condition that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel

then $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Taking option (C) and applying the above condition on it and in the given equation.

or $$\frac{3}{-12} = \frac{-2}{8} \neq \frac{5}{7}$$

Top 10

Most Important Questions

Q. What is the value of k such that the following pair of equations have infinitely many solutions ?

$x-2y = 3$ and $-3x + ky = -9$.

(A) -6

(B) -3

(C) 3

(D) 6

SOLUTION

. **Option (D) is correct.**

Explanation: Given equations are:

$$x - 2y - 3 = 0$$

and $$-3x + ky + 9 = 0$$

Here, $\frac{a_1}{a_2} = \frac{1}{-3}, \frac{b_1}{b_2} = \frac{-2}{k}$ and $\frac{c_1}{c_2} = \frac{-3}{9}$

For infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ For $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{1}{-3} = \frac{-2}{k} \Rightarrow k = 6$$

and for $$\frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{-2}{k} = \frac{-3}{9}$$

$\Rightarrow 3k = 18 \Rightarrow k = 6$

Hence, the value of k is 6.

Top 10

Most Important Questions

Q.If a pair of linear equations given by $a_1x + b_1y + c_1 = 0$ and $a_2x + by + c_2 = 0$ has a unique solution, then which of the following is true?

(A) $a_1a_2 = b_1b_2$

(C) $a_1b_2 \neq a_2b_1$

(B) $a_1b_2 = a_2b_1$

(D) $a_1a_2 \neq b_1b_2$

SOLUTION

Option (B) is correct.

Explanation: For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\therefore \quad a_1b_2 \neq a_2b_1$$

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If the pair of linear equations $3x + y = 3$ and $6x + ky = 8$ does not have a solution, then the value of $k = 2$.

Reason (R): If the pair of linear equations $x+y-4 = 0$ and $2x + ky = 3$ does not have a solution, then the value of $k = 2$.

SOLUTION

. **Option (B) is correct.**

Explanation: In case of assertion:

Given equations are:

$$3x + y - 3 = 0 \quad \text{...(i)}$$

and $$6x + ky - 8 = 0 \quad \text{...(ii)}$$

Comparing eq. (i) with $a_1x + b_1y + c_1 = 0$ and eq. (ii) with $a_2x + b_2y + c_2 = 0$, we get,

$a_1 = 3, a_2 = 6, b_1 = 1, b_2 = k, c_1 = -3$ and $c_2 = -8$

Since, given equations has no solution.

So, $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{6} = \frac{1}{k} \neq \frac{-3}{-8}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} \neq \frac{3}{8}$$

Either $\frac{1}{2} = \frac{1}{k}$ or $\frac{1}{k} \neq \frac{3}{8}$

$$\Rightarrow \quad k = 2 \text{ or } k \neq \frac{-8}{3}$$

Hence, the value of k is 2.

Thus, assertion is true

In case of reason:

Given equations:

$$x + y - 4 = 0$$

and $$2x + ky - 3 = 0$$

Here $$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{k}$$

and $$\frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$$

$\because$ System has no solution.

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$

$$\Rightarrow \quad k = 2 \text{ or } k \neq \frac{3}{4}$$

Hence, the value of k is 2.

Thus, reason is true.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): For all real values of c, the pair of equations $x - 2y = 8$ and $5x - 10y = c$ have a unique solution.

Reason (R): Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, $12x + 9y = 5$.

SOLUTION

. **Option (D) is correct.**

Explanation: In case of assertion:

$$x - 2y = 8 \quad \text{...(i)}$$

$$5x - 10y = c \quad \text{...(ii)}$$

$$\therefore \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, so system of linear equations can never have unique solution.

$\therefore$ Assertion is false.

In case of reason:

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{-14}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} \Rightarrow \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

$\therefore$ Reason is true.

Hence, assertion is false but reason is true.

Q. If $3x + 4y : x + 2y = 9 : 4$, then $3x + 5y : 3x - y$ is equal to

[Board 2022]

a) $4 : 1$

b) $1 : 4$

c) $7 : 1$

d) $1 : 7$

Solution

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$

Hence, $12x+16y = 9x+18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

Substituting $x = \frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

Thus (c) is correct option.

Q. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is

[Board 2022]

a) 2

b) 3

c) 5

d) 15

Solution

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y + 3 \qquad ...(1)$$

and $$\frac{x-1}{y-1} = 7 \Rightarrow x = 7y - 6 \qquad ...(2)$$

Solving (1) and (2), we have $x = 15$, $y = 3$,
Thus (d) is correct option.

Q. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel

$3x - y - 5 = 0$

$6x - 2y - p = 0$

[Board 2022]

a) all real values except 10

b) 10

c) 5/2

d) 1/2

Solution

We have, $3x - y - 5 = 0$

and $6x - 2y - p = 0$

Here, $a_1 = 3,\ b_1 = -1,\ c_1 = -5$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -p$

Since given lines are parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{-1}{-2} \neq \frac{-5}{-p}$$

$$p \neq 5 \times 2 \Rightarrow p \neq 10$$

Q. In a number of two digits, unit’s digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is

[Board 2022]

a) 36

b) 63

c) 48

d) 84

Solution

Let x be units digit and y be tens digit, then number will be $10y + x$

Then, $$x = 2y \quad ...(1)$$

If 36 be added to the number, the digits are reversed, i.e number will be $10x + y$.

$$10y + x + 36 = 10x + y$$

$$9x - 9y = 36$$

$$x - y = 4 \quad ...(2)$$

Solving (1) and (2) we have $x = 8$ and $y = 4$.

Thus number is 48.

Thus (c) is correct option.

Q. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is

[Board 2020]

a) -2

b) $\neq 2$

c) 3

d) 2

Solution

We have $x + y - 4 = 0$

and $2x + ky - 3 = 0$

Here, $\frac{a_1}{a_1} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{k}$ and $\frac{c_1}{c_2} = \frac{-4}{-3} = \frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$

$$k = 2 \text{ and } k \neq \frac{3}{4}$$

Thus (d) is correct option.

Q. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has

[Board 2020]

a) a unique solution

b) exactly two solutions

c) infinitely many solutions

d) no solution

Solution

Given,equations are

$$x + 2y + 5 = 0$$

and $$-3x - 6y + 1 = 0$$

Here, $a_1 = 1, b_1 = 2,\ c_1 = 5$

and $a_2 = -3,\ b_2 = -6,\ c_2 = 1$

Now $$\frac{a_1}{a_2} = -\frac{1}{3},\ \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3},\ \frac{c_1}{c_2} = \frac{5}{1}$$

Now, we observe that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.
Thus (d) is correct option.

Q. If a pair of linear equations is consistent, then the lines will be

a. parallel

b. always coincident

c. intersecting or coincident

d. always intersecting

Solution

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ [coincident or dependent]

Thus (c) is correct option.

Q. Aruna has only ₹1 and ₹2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹75, then the number of ₹1 and ₹2 coins are, respectively

a. 35 and 15

b. 35 and 20

c. 15 and 35

d. 25 and 25

Solution

Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

Now, by given conditions,

$$x + y = 50 \qquad ...(1)$$

Also, $x \times 1 + y \times 2 = 75$

$$x + 2y = 75 \qquad ...(2)$$

Subtracting equation (1) form equation (2), we get

$$(x + 2y) - (x + y) = 75 - 50$$

$$y = 25$$

From equation (i), $x = 75 - 2x(25)$

Then, $x = 25$

Thus (d) is correct option.

Q. Assertion : Pair of linear equations : $9x + 3y + 12 = 0$, $8x + 6y + 24 = 0$ have infinitely many solutions.

Reason : Pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

From the given equations, we have

$$\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2} \text{ i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Q. Assertion : $x + y - 4 = 0$ and $2x + ky - 4 = 0$ has no solution if $k = 2$.

Reason : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are consistent if $\frac{a_1}{a_2} \neq \frac{k_1}{k_2}$.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

For assertion, given equation has no solution if

$$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3} \text{ i.e. } \frac{4}{3}$$

$$k = 2\left[\frac{1}{2} \neq \frac{4}{3} \text{holds}\right]$$

Assertion is true.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
Thus (b) is correct option.

Top 10

Most Important Questions

Q. Let p be a prime number. The quadratic equation having its roots as factors of p is

(A) $x^2 - px + p = 0$ 2

(B) $x^2 - (p + 1)x + p = 0$

(C) $x^2 + (p + 1)x + p = 0$

(D) $x^2 - px + p + 1 = 0$

[CBSE SQP, 2022-23]

SOLUTION

. **Option (B) is correct.**

Explanation: Factors of $p = p \times 1$

$\therefore$ Roots are p and 1.

The quadratic equation is:

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$\Rightarrow \quad x^2 - (p + 1)x + p = 0$

Top 10

Most Important Questions

Q.Which of the following is not a quadratic equation?

(A) $2(x-1)^2=4x^2-2x + 1$

(B) $2x-x^2 = x^2 +5$

(C) $(\sqrt{2}x + \sqrt{3})^2 +x^2= 3x^2-5x$

(D) $(x^2+2x)^2 = x^2 + 3 + 4x^3$

SOLUTION

C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

By using algebraic identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$

By grouping,

$3x^2 - 3x^2 + 2\sqrt{6}x + 5x + 3 = 0$

$5x + 2\sqrt{6}x + 3 = 0$

The degree of the equation is 1

Therefore, $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ is not a quadratic equation.

Top 10

Most Important Questions

Q. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) two distinct real roots

(B) two equal real roots

(C) no real root

(D) more than 2 real roots

SOLUTION

. Option (C) is correct.

Explanation: $2x^2 - \sqrt{5}x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$

$a = 2, b = -\sqrt{5}, c = 1$

Discriminant, $D = b^2 - 4ac = (-\sqrt{5})^2 - 4(2)(1)$

$= 5 - 8 = -3 < 0$

Since D (*i.e.*, $b^2 - 4ac$) < 0

Therefore, the equation has no real root.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): One solution of the quadratic equation $(x - 1)^2 - 5(x-1) - 6 = 0$ is 0.

Reason (R): Other solution of the equation $(x-1)^2 - 5(x-1) - 6 = 0$ is 7.

SOLUTION

. Option (A) is correct.

Explanation: In case of assertion:

Given, $(x-1)^2 - 5(x-1) - 6 = 0$

$$\Rightarrow \quad x^2 - 2x + 1 - 5x + 5 - 6 = 0$$

$$\Rightarrow \quad x^2 - 7x + 6 - 6 = 0$$

$$\Rightarrow \quad x^2 - 7x = 0$$

$$\Rightarrow \quad x(x-7) = 0$$

$$\therefore \quad x = 0 \text{ or } 7$$

$\therefore$ Assertion is true.

In case of reason:

From the above solution, the other solutions of given equation is 7.

$\therefore$ Reason is also true.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): Every quadratic equation has exactly one root.

Reason (R): Every quadratic equation always has degree two.

SOLUTION

Option (D) is correct.

Explanation: In case of assertion:

Since, a quadratic equation has only two roots.

$\therefore$ Assertion is false.

In case of reason:

Every quadratic equation always has degree two which gives only two roots (or solutions)

$\therefore$ Reason is true.

Hence, assertion is false but reason is true.

Top 10

Most Important Questions

Q. The first term of A.P. is p and the common difference is q, then its 10th term is

(A) q+ 9p

(C) p +9q

(B) p-9q

(D) 2p +9q [CBSE Delhi Set-I, 2020]

SOLUTION

l. Option (C) is correct.

Explanation: $a = p$ and $d = q$ (given)

$\therefore$ 10^{th} term $= a + (10 - 1)d$

$= p + 9q$

Top 10

Most Important Questions

Q. The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P., is

(A) 6

(C) 18

(B) -6

(D) -18

[CBSE Delhi Set-I, 2020]

SOLUTION

Top 10

Most Important Questions

Q..If the first term of an A.P. is -5 and the common difference is 2, then the sum of the first 6 terms is:

(A) 0

(C) 6

(B) 5

(D) 15

SOLUTION

Explanation: In the given A.P.,

$$a = -5 \text{ and } d = 2$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow \quad S_6 = \frac{6}{2}\left[2\times(-5) + (6-1)\times 2\right]$$

$$= 0$$

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If the second term of an A.P., is 13 and the fifth term is 25, then its 7th term is 33.

Reason (R): If the common difference of an A.P. is 5, then a18a13 is 25.

SOLUTION

. **Option (B) is correct.**

Explanation: In case of assertion:

In the given A.P., $a_2 = 13$ and $a_5 = 25$

$$a + d = 13$$

$$a + 4d = 25$$

Solving these equations, we get $a = 9$ and $d = 4$

Thus,

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad a_7 = 9 + (7-1)4 = 33$$

$\therefore$ Assertion is true.

In case of reason:

In the given A.P., $d = 5$ Thus,

$a_{18} - a_{13} = a + 17d - a - 12d = 5d = 25$

$\therefore$ Reason is true.

Hence, both assertion and reason are true but reason is not the correct explanation for assertion.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If nth term of an A.P. is (2n + 1), then the sum of its first three terms is 15.

Reason (R): The sum of first 16 terms of the A.P. 10, 6, 2, ... is 420.

SOLUTION

;. **Option (C) is correct.**

Explanation: In case of assertion:

$\because \quad a_n = (2n + 1)$

$\therefore \quad a_1 = 2 \times 1 + 1 = 3$

$l = a_3 = 2 \times 3 + 1 = 7$

Since, $S_n = \frac{n}{2}[a + l]$

Hence, $S_3 = \frac{3}{2}[3 + 7]$

$S_3 = 15$

$\therefore$ Assertion is true.

In case of reason:

Here, $a = 10$, $d = 6 - 10 = -4$ and $n = 16$

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{16} = \frac{16}{2}[2 \times 10 + (16 - 1)(-4)]$

$= 8[20 + 15 \times (-4)]$

$= 8[20 - 60]$

$= 8 \times (-40)$

$= -320$

$\therefore$ Reason is false.

Hence, assertion is true but reason is false.

Top 10

Most Important Questions

Q. In $\triangle ABC$ and $\triangle DEF$, $\angle F = \angle C$, $\angle B = \angle E$ and $AB = \frac{1}{2} DE$. Then the two triangles are

(A) Congruent, but not similar

(B) Similar, but not congruent

(C) Neither congruent nor similar

(D) Congruent as well as similar

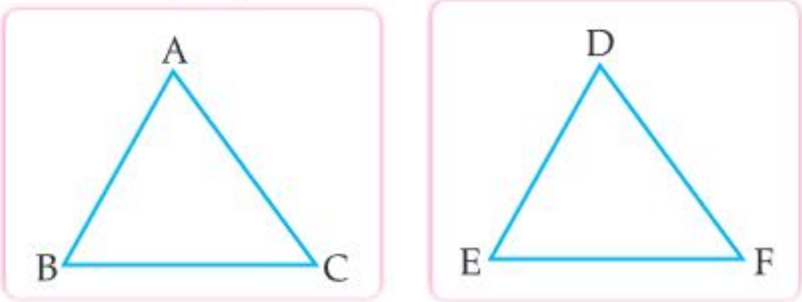
[CBSE, Board Term-I, 2021]

SOLUTION

[CBSE, Board Term I, 2021]

Sol. Option (B) is correct.

Explanation: According to the definition of similarity of two triangles, "Two triangles are similar when their corresponding angles are equal and the sides are in proportion".



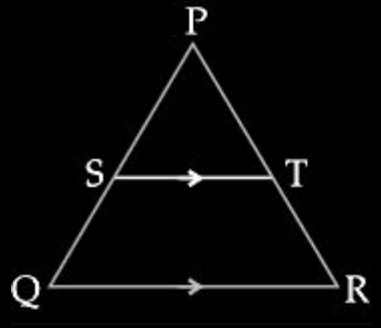
According to the question,

$\angle F = \angle C$ and $\angle B = \angle E$

Top 10

Most Important Questions

Q. In the following figure, ST || QR, point S divides PQ in the ratio 4: 5. If ST = 1.6 cm, what is the length of QR?

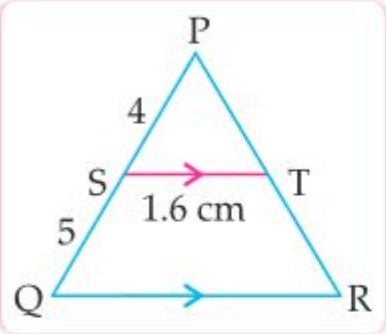


(A) 0.71 cm
(B) 2 cm
(C) 3.6 cm
(D) cannot be calculated from the given data.

SOLUTION

Sol. Option (C) is correct.

Explanation:



$$ST \parallel QR \qquad \text{(Given)}$$

$$\frac{PS}{PQ} = \frac{ST}{QR} \qquad \text{(Corr. B.P.T.)}$$

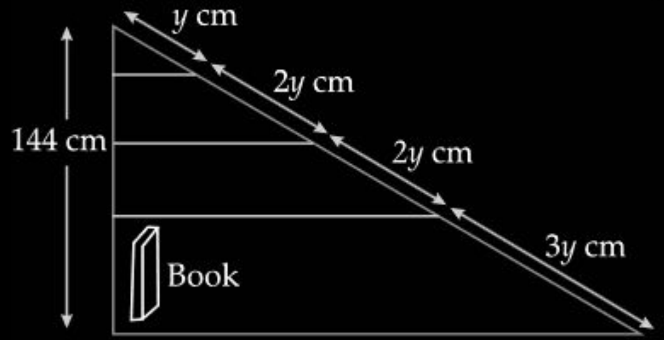
$$\Rightarrow \quad \frac{4}{9} = \frac{1.6}{QR}$$

$$\therefore \quad QR = \frac{9}{4} \times 1.6 = 3.6 \text{ cm.}$$

Top 10

Most Important Questions

Q..Leela has a triangular cabinet that fits under his staircase. There are four parallel shelves as shown below. The total height of the cabinet is 144 cm. What is the maximum height of a book that can stand upright on the bottom-most shelf?



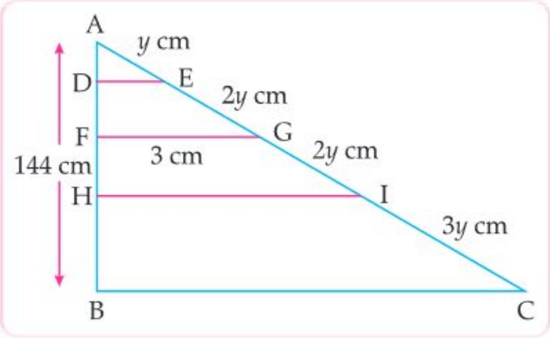
(A) 18 cm
(C) 54 cm
(B) 36 cm
(D) 86.4 cm

SOLUTION

[Q. 5, CBSE CFT Q]

Sol. Option (C) is correct.

Explanation:



1

In ΔABC

$$HI \parallel BC \qquad \text{(Given)}$$

$$\therefore \quad \frac{AB}{HB} = \frac{AC}{IC}$$

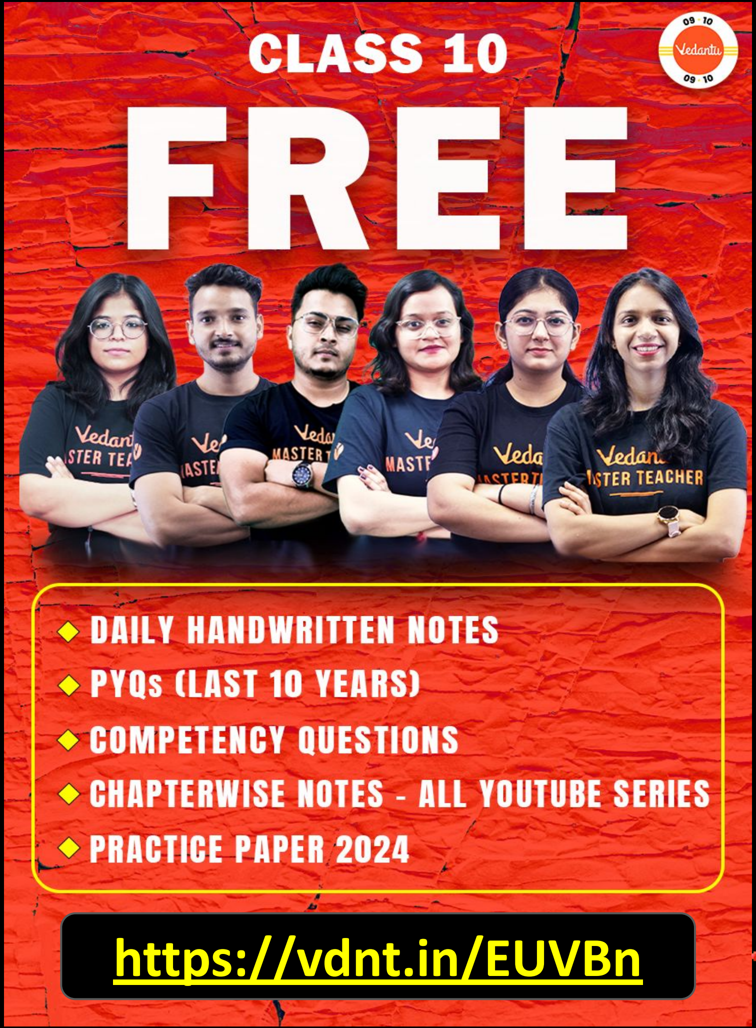
(According to basic proportional theorem)

$$\frac{144}{HB} = \frac{8y}{3y}$$

$$\frac{144 \times 3}{8} = HB$$

$$HB = 54 \text{ cm}$$

Thus, the maximum height of a book is 54 cm.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DAILY HANDWRITTEN NOTES
PYQs (LAST 10 YEARS)
COMPETENCY QUESTIONS
CHAPTERWISE NOTES - ALL YOUTUBE SERIES
PRACTICE PAPER 2024
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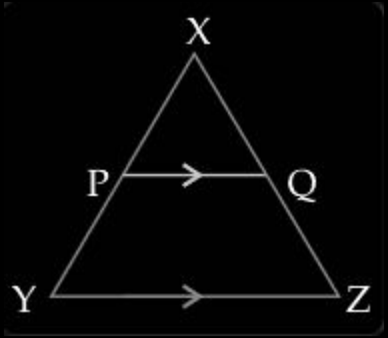
Top 10

Most Important Questions



Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): In the following figure, PQ || YZ, point P divides XY in the ratio 3: 4. If PQ = 1.5 cm. The length of YZ is 3.5 cm.

Reason (R): The ratio of XQ and XZ is 3: 7.

SOLUTION

. Option (A) is correct.

Explanation: For assertion:

$$PQ \parallel YZ \quad \text{(Given)}$$

$$\therefore \quad \frac{XP}{XY} = \frac{PQ}{YZ} \quad \text{(From BPT)}$$

$$\Rightarrow \quad \frac{3}{7} = \frac{1.5}{YZ}$$

$$\therefore \quad YZ = \frac{7 \times 1.5}{3}$$

$$= 3.5 \text{ cm.}$$

So, assertion is true.

For reason:

$$\because \quad \frac{XQ}{XZ} = \frac{PQ}{YZ} \quad \text{(From BPT)}$$

$$\Rightarrow \quad \frac{XQ}{XZ} = \frac{1.5}{3.5}$$

[From $YZ = 3.5$ proved above]

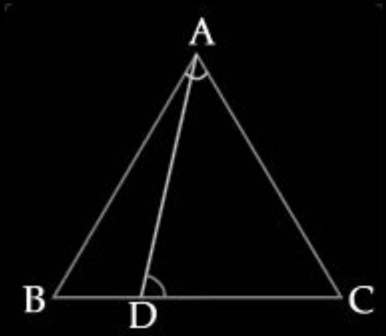
Hence, $XQ : XZ = 3 : 7$.

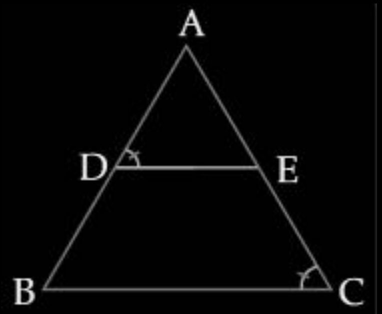
Reason is also true.

So, both A and R are true and R is the correct explanation of A.

Top 10

Most Important Questions





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(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If D is a point on the side BC of a triangle ABC such that ∠ADC= ∠BAC, CA^2 = CB × CD.

Reason (R): In the figure given below, if ∠D = ∠C then△ ADE □△ACB.

SOLUTION

Ans. Option (B) is correct.

Explanation:

For assertion:

In ΔABC and ΔADC,

$$\angle BAC = \angle ADC \quad \text{(Given)}$$

$$\angle BCA = \angle ACD \quad \text{(Common angle)}$$

By AA similarity, $\Delta ABC \sim \Delta ADC$

Thus, $$\frac{CA}{CD} = \frac{BC}{CA}$$

$$\Rightarrow \quad CA^2 = CB \times CD$$

So, assertion is true.

For reason:

In ΔADE and ΔACB, we have

$$\angle ADE = \angle ACB \quad \text{(Given)}$$

$$\angle DAE = \angle CAB \quad \text{(Common angle)}$$

By AA similarity, we get ΔADE ~ ΔACB.

∴ Reason is also, true.

So, both A and R are true and R is not the correct explanation of A.

Q. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3), then the value of x is

a) 0

b) 1

c) 2

d) 3

Solution

Since, A and B lie on the circle having centre O.

$$\begin{aligned} OA &= OB \\ \sqrt{(4-2)^2+(3-3)^2} &= \sqrt{(x-2)^2+(5-3)^2} \\ 2 &= \sqrt{(x-2)^2+4} \\ 4 &= (x-2)^2+4 \\ (x-2)^2 &= 0 \Rightarrow x = 2 \end{aligned}$$

Thus (c) is correct option.

Q. The ratio in which the point (2, y) divides the join of (- 4, 3) and (6, 3), hence the value of y is

a) $2 : 3 , y = 3$

b) $3 : 2 , y = 4$

c) $3 : 2 , y = 3$

d) $3 : 2 , y = 2$

Solution

Let the required ratio be $k:1$

Then, $$2 = \frac{6k - 4(1)}{k+1}$$

or $$k = \frac{3}{2}$$

The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $$y = \frac{3(3)+2(3)}{3+2} = 3$$

Thus (c) is correct option.

Q. The point P on x -axis equidistant from the points A(- 1, 0) and B(5, 0) is (a)

[Board 2020 OD Standard]

a (2, 0)

b (0, 2)

c (3, 0)

d (- 3, 5)

Solution

Let the position of the point P on x-axis be $(x, 0)$, then

$$\begin{aligned} PA^2 &= PB^2 \\ (x+1)^2 + (0)^2 &= (5-x)^2 + (0)^2 \\ x^2 + 2x + 1 &= 25 + x^2 - 10x \\ 2x + 10x &= 25 - 1 \\ 12x &= 24 \Rightarrow x = 2 \end{aligned}$$

Hence, the point $P(x, 0)$ is (2, 0).
Thus (a) is correct option.

Alternative :

You may easily observe that both point $A(-1, 0)$ and $B(5, 0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1, 0)$ and $B(5, 0)$.

$$x = \frac{-1+5}{2} = 2$$

Q. The coordinates of the point which is reflection of point (-3, 5) in x -axis are

[Board 2020 OD Standard]

a. (3, 5)

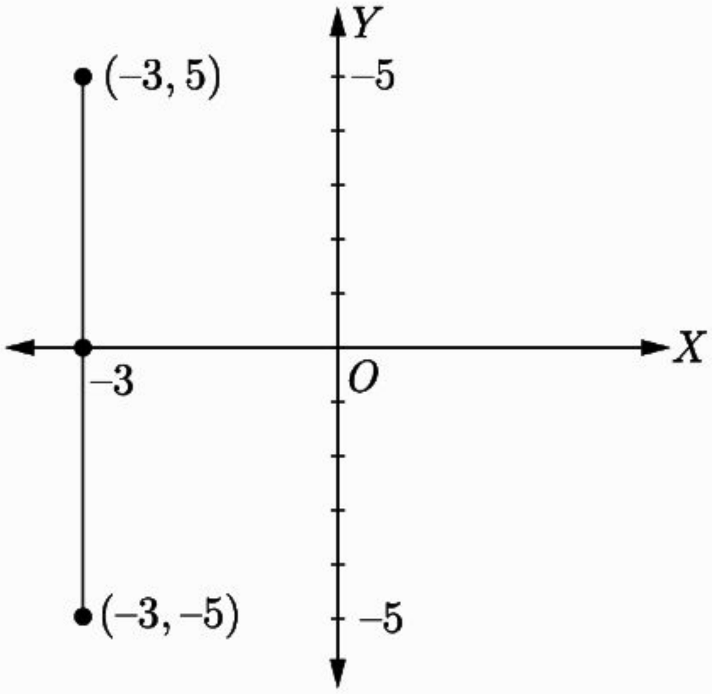
b. (3, -5)

c. (-3, -5)

d. (- 3, 5)

Solution

The reflection of point $(-3, 5)$ in x- axis is $(-3, -5)$.



Thus (c) is correct option.

Q. If the point P (6, 2) divides the line segment joining A(6, 5) and B(4, y) in the ratio 3 : 1 then the value of y is

[Board 2020 OD Standard]

a) 4

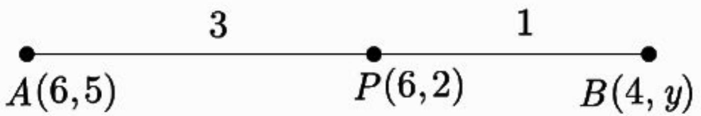
b) 3

c) 2

d) 1

Solution

As per given information in question we have drawn the figure below,



Here, $x_1 = 6,\ y_1 = 5$

and $x_2 = 4\ \ y_2 = y$

Now
$$y = \frac{my_2 + ny_1}{m + n}$$

$$2 = \frac{3 \times y + 1 \times 5}{3 + 1}$$

$$2 = \frac{3y + 5}{4}$$

$$3y + 5 = 8$$

$$3y = 8 - 5 = 3 \Rightarrow y = 1$$

Thus (d) is correct option.

Q. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$ is

[Board 2020 Delhi Standard]

a) $a^2 + b^2$

b) $a^2 - b^2$

c) $\sqrt{a^2 + b^2}$

d) $\sqrt{a^2 - b^2}$

Solution

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos\theta$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2$$
$$= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta +$$
$$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$
$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$
$$= a^2 \times 1 + b^2 \times 1 = a^2 + b^2$$

Thus $d^2 = a^2 + b^2$

$$d = \sqrt{a^2 + b^2}$$

Therefore (c) is correct option.

Q. The point which divides the line segment joining the points (8, -9) and (2, 3) in the ratio 1 : 2 internally lies in the

a) I quadrant

b) II quadrant

c) III quadrant

d) IV quadrant

Solution

We have $x_1 = 8$, $y_1 = -9$, $x_2 = 2$ and $y_2 = 3$.

and $m_1 : m_2 = 1:2$

Let the required point be $P(x, y)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 8}{1 + 2} = 6$$

and $$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 3 + 2(-9)}{1 + 2} = -5$$

Thus $(x, y) = (6, -5)$ and this point lies in IV quadrant.

Thus (d) is correct option.

Q. If $A(m/3, 5)$ is the mid-point of the line segment joining the points $Q(-6, 7)$ and $R(-2, 3)$, then the value of m is

[Board 2020 SQP Standard]

a. -12

b. -4

c. 12

d. -6

Solution

Given points are $Q(-6,7)$ and $R(-2,3)$

Mid point $\quad A(\frac{m}{3},\ 5) = \left(\frac{-6-2}{2},\ \frac{7+3}{2}\right)$

$$= (-4, 5)$$

Equating, $\quad \frac{m}{3} = -4 \Rightarrow m = -12$

Thus (a) is correct option.

Q. If three points (0, 0), (3, $\sqrt{3}$) and (3, λ) form an equilateral triangle, then λ equals

a) 2

b) -3

c) -4

d) None of these

Solution

Let the given points are $A(0,0)$, $B(3,\sqrt{3})$ and $C(3,\lambda)$.
Since, ΔABC is an equilateral triangle, therefore

$$\begin{aligned} AB &= AC \\ \sqrt{(3-0)^2+(\sqrt{3}-0)^2} &= \sqrt{(3-0)^2+(\lambda-0)^2} \\ 9+3 &= 9+\lambda^2 \\ \lambda^2 &= 3 \Rightarrow \lambda = \pm\sqrt{3} \end{aligned}$$

Thus (d) is correct option.

Q. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is

a) 4 only

b) ±4

c) -4

d) 0

Solution

According to the question, the distance between the points $(4, p)$ and $(1, 0)$ is 5.

i.e.,

$$\sqrt{(1-4)^2+(0-p)^2} = 5$$

$$\sqrt{(-3)^2+p^2} = 5$$

$$\sqrt{9+p^2} = 5$$

Squaring both the sides, we get,

$$9+p^2 = 25$$

$$p^2 = 16 \Rightarrow p = \pm 4$$

Hence, the required value of p is ± 4.

Thus (b) is correct option.

Q. Assertion: The value of y is 6, for which the distance between the points P(2, -3) and Q(10, y) is 10.

Reason : Distance between two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

$$
\begin{aligned}
PQ &= 10 \\
PQ^2 &= 100 \\
(10-2)^2+(y+3)^2 &= 100 \\
(y+3)^2 &= 100-64 = 36 \\
y+3 &= \pm 6 \\
y &= -3 \pm 6 \\
y &= 3, -9
\end{aligned}
$$

Assertion (A) is false but reason (R) is true.
Thus (s) is correct option.

Q. Assertion : The point (0, 9) lies on y-axis.

Reason(R): The x coordinate of the point on the y-axis is zero.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We know that the if the point lies on y-axis, its x-coordinate is 0. So, Reason is correct.

The x co-ordinate of the point (0, 9) is zero.

So, Point (0, 9) lies on y -axis.

So, Assertion is also correct

Correct option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Q. Assertion : C is the mid-point of PQ if P is (4, x), C is (y, -1) and Q is (-2, 4), then x and y respectively are -6 and 1.

Reason(R): The mid-point of the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We know that the mid-point of the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So, Reason is correct.

Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$

We have, $\frac{4-2}{2} = y \Rightarrow y = 1$

and $\frac{x+4}{2} = -1 \Rightarrow x + 4 = -2$

$\Rightarrow x = -6$

So, Assertion is correct

Correct option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of asssertion (A)

Top 10

Most Important Questions

Q. Three vertices of a parallelogram ABCD are A(1, 4), B(-2, 3) and C(5, 8). The ordinate of the fourth vertex D is

(A) 8

(B) 7

(C) 9

(D) 6

[CBSE, Board Term-I, 2021]

SOLUTION

Sol. Option (B) is correct.

Explanation: Let A(1, 4), B(–2, 3), C(5, 8) and D(a, b) are the vertices of a parallelogram.

Midpoint of diagonal AC

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1+5}{2}, \frac{4+8}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{12}{2}\right) = (3, 6)$$

Midpoint of diagonal BD

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+a}{2}, \frac{3+b}{2}\right)$$

The diagonals of the parallelogram bisect each other. The diagonals share same mid-point.

$$\therefore \qquad (3, 6) = \left(\frac{-2+a}{2}, \frac{3+b}{2}\right)$$

On comparing both sides, we get

$$3 = \frac{-2+a}{2} \text{ and } 6 = \frac{3+b}{2}$$

In the question, value of ordinate is asked,

$$6 = \frac{3+b}{2}$$

$$12 = 3 + b$$

or $\qquad b = 9.$

Top 10

Most Important Questions

Q. The line segment joining the points P(-3, 2) and Q(5, 7) is divided by the Y-axis in the ratio

(A) 3:1

(B) 3:2

(C) 3:4

(D) 3:5 [CBSE, Board Term-I, 2021]

SOLUTION

Concept/Formula to be used:

x coordinate of any point at y axis is 0

coordinate of point dividing point (x_1, y_1) and (x_2, y_2) in m : n ratio is given by:

$$\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$$

Substitute $\mathbf{x_1 = -3}$, $\mathbf{x_2 = 5}$ and x coordinate = 0 and solve for m and n

$$\frac{m(5) + n(-3)}{m+n} = 0$$

5m - 3n = 0

=> 5m = 3n

=> m/n = 3/5

=> **m : n = 3 : 5**

Correct option is **d) 3: 5**

Top 10

Most Important Questions

Q△ ABC is a triangle such that AB : BC = 1 : 2. Point A lies on the Y-axis and the coordinates of B and C are known.

Which of the following formula can Definitely be used to find the coordinates of A?

(i) Section formula

(ii) Distance formula

(A) only (i)

(C) both (i) and (ii)

(B) only (ii)

(D) neither (i) or (ii)

SOLUTION

Sol. Option (B) is correct.

Explanation: As A lies on the y-axis therefore co-ordinates of A will be $(0, y)$

So, we can find value of y-coordinate by using distance formula.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): The ordinate of a point A on y-axis is 5 and B has co-ordinates (-3, 1). Then the length of AB is 5 units.

Reason (R): The point A(2, 7) lies on the perpendicular bisector of line segment joining the points P(6, 5) and Q(0, -4).

SOLUTION

. Option (C) is correct.

Explanation: In case of assertion:

Here, A → (0 , 5) and B → (– 3 , 1)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-3-0)^2 + (1-5)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

∴ Assertion is true.

In case of reason:

If A(2, 7) lies on perpendicular bisector of P(6, 5) and Q(0, –4), then

$$AP = AQ$$

∴ By using Distance Formula,

$$AP = \sqrt{(6-2)^2 + (5-7)^2}$$

$$= \sqrt{(4)^2 + (-2)^2}$$

$$= \sqrt{20}$$

And $$AQ = \sqrt{(0-2)^2 + (-4-7)^2}$$

$$= \sqrt{(-2)^2 + (-11)^2}$$

$$= \sqrt{125}$$

As, $AP \neq AQ$

Therefore, A does not lie on the perpendicular bisector of PQ.

∴ Reason is false.

Hence, assertion is true but reason is false.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

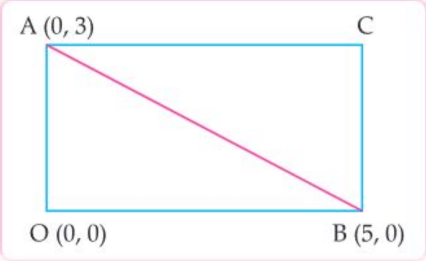
Assertion (A): AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). The length of its diagonal is $\sqrt{34}$ units.

Reason (R): The distance between AO is 3 units.

SOLUTION

Ans. Option (B) is correct.

Explanation: In case of assertion,



$\therefore$ Distance between the point A(0, 3) and B(5, 0)

$$= \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34} \text{ units.}$$

So, assertion is true.

In case of reason,

Distance between the points A(0, 3) and O(0, 0)

$$= \sqrt{(0-0)^2 + (0-3)^2}$$

$$= \sqrt{0+9}$$

$$= \sqrt{9}$$

$$= 3 \text{ units}$$

So, reason is also true.

Q. Which of the following statement is false?

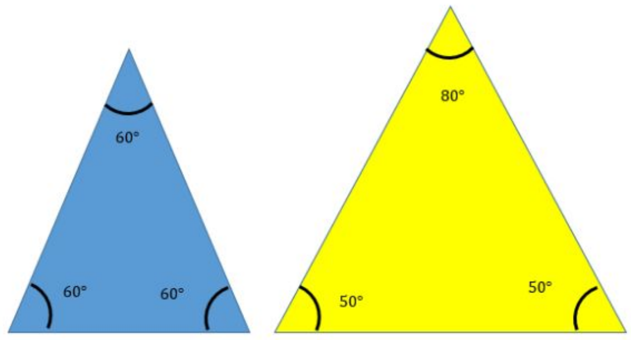
a. All isosceles triangles are similar.

b. All equilateral triangles are similar.

c. All circles are similar.

d. None of the above

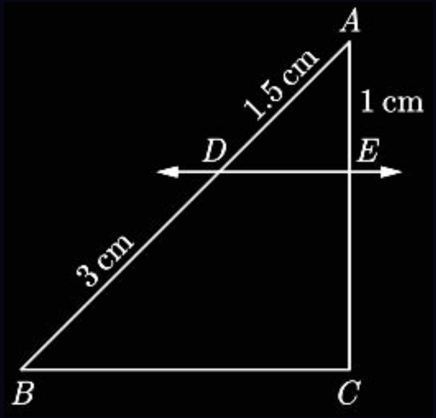
Solution



If these isosceles triangles are similar, then they must follow either one of the AA or SAS criterion but they cannot because none of the angles is equal to one another. Hence, our assumption was incorrect.

Hence, option (A) is false.

Q. In the given figure, DE || BC. The value of EC is



a 1.5 cm

b 3 cm

c 2 cm

d 1 cm

Solution

Given, in △ ABC, DE∥BC

∴ AD/DB = AE/EC (Using Basic proportionality theorem)

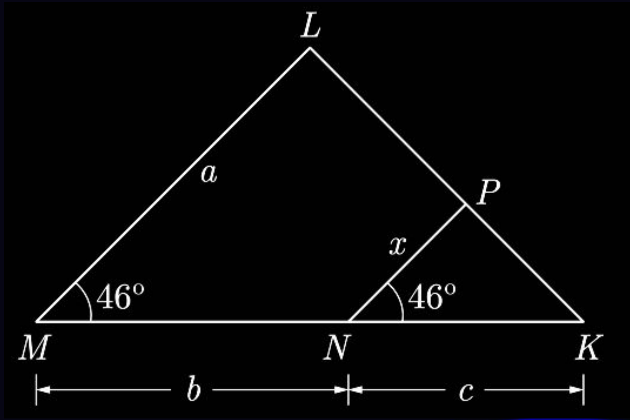
⇒1.5/3 = 1/EC

⇒EC = 3/1.5

EC = 3×10/15 = 2 cm

Hence, EC = 2 cm.

Q. In the given figure, x is



a $\dfrac{ab}{a+b}$

b $\dfrac{ac}{b+c}$

c $\dfrac{bc}{b+c}$

d $\dfrac{ac}{a+c}$

Solution

In Δs LMK and ONK,

∠KML = ∠ONK = 46°

K = ∠K (Common)

∴ ΔLMK ~ ΔONK (AA similarity)

$$\Rightarrow \frac{KM}{KN} = \frac{LM}{ON} \Rightarrow \frac{b+c}{c} = \frac{a}{x} \Rightarrow x = \frac{ac}{b+c}$$

Q. 🔺ABC is an equilateral triangle with each side of length 2p. If AD ⊥ BC = then the value of AD is

a) √3

b) √3p

c) 2p

d) 4p

Solution

$\triangle ABC$ is an equilateral triangle with each side of length $2p$.

That means, $AB = BC = CA = 2p$

Also, $AD \perp BC$

As we know, in an equilateral triangle, the perpendicular drawn from a vertex to the opposite side bisects the opposite side.

So, $BD = DC = \frac{1}{2}BC = p$

Now, triangle $\triangle ADB$ is a right triangle.

By Pythagoras theorem,

$$AD^2 + BD^2 = AB^2$$

$$AD^2 + p^2 = (2p)^2$$

$$AD^2 = 4p^2 - p^2$$

$$AD^2 = 3p^2$$

$$AD = p\sqrt{3}$$

Therefore, the value of $AD = \sqrt{3}p$.

Q. It is given that, $\triangle ABC \sim \triangle EDF$ such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm then the sum of the remaining sides of the triangles is

a) 23.05 cm

b) 16.8 cm

c) 6.25 cm

d) 24 cm

Solution

Consider AB/ED = AC/EF

5/12 = 7/EF

By cross multiplication,

$EF = (7 \times 12)/5$

EF = 16.8cm

Now, consider AB/ED = BC/DF

5/12 = BC/15

$BC = (5 \times 15)/12$

BC = 75/12

BC = 6.25.

Q. Assertion : In the △ ABC , AB = 24 cm, BC = 10 cm and AC = 26 cm, then △ ABC is a right angle triangle.

Reason: If in two triangles, their corresponding angles are equal, then the triangles are similar.

Solution

We have, $AB^2 + BC^2 = (24)^2 + (10)^2$

$$= 576 + 100$$

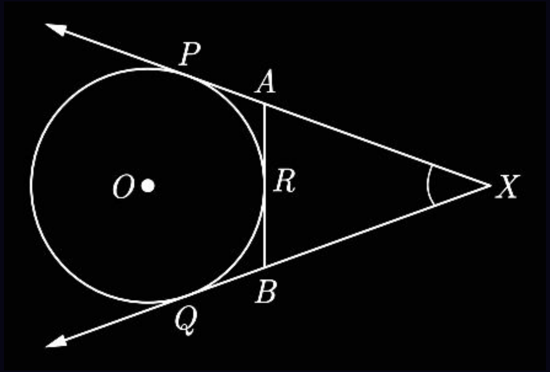
$$= 676 = AC^2$$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.



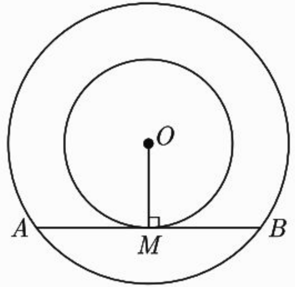
Q. Assertion : In the given figure, XA + AR = XB + BR, where XP, XQ & AB are tangents.

Reason : In two concentric circle, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

Solution

We have,
$$\begin{aligned} XP &= XQ \\ XA + AP &= XB + BQ \\ XA + AR &= XB + BR \end{aligned}$$
$$[PA = AR \text{ and } BQ = BR]$$

(The length of tangents drawn from in external point are equal)



We have two concentric circles (shown in fig. 8.17 b) O is the centre of concentric circles and AB is the tangent

$$\begin{aligned} OM &\perp AB \\ AM &= MB \end{aligned}$$

(Perpendicular from centre O to the chord AB bisect the chord AB)

Q. Assertion: The value of y is 6, for which the distance between the points P(2, -3) and Q(10, y) is 10.

Reason : Distance between two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution

$$
\begin{aligned}
PQ &= 10 \\
PQ^2 &= 100 \\
(10-2)^2+(y+3)^2 &= 100 \\
(y+3)^2 &= 100-64 = 36 \\
y+3 &= \pm 6 \\
y &= -3 \pm 6 \\
y &= 3, -9
\end{aligned}
$$

Assertion (A) is false but reason (R) is true.
Thus (d) is correct option.

Top 10

Most Important Questions

Q. Given that $\sec\theta = \sqrt{2}$, the value of

$$\frac{1+\tan\theta}{\sin\theta}$$ Is

(A) $2\sqrt{2}$

(B) $\sqrt{2}$

(C) $3\sqrt{2}$

(D) 2

[CBSE, Board Term-I, 2021]

SOLUTION

Ans. Option (A) is correct.

Explanation: It is given that

$$\sec\theta = \sqrt{2} \quad \text{...(i)}$$

Also, $\sec 45^\circ = \sqrt{2}$ ...(ii)

From (i) and (ii), we get

$$\theta = 45^\circ$$

Put value of θ in $\frac{1+\tan\theta}{\sin\theta}$,

$$\Rightarrow \quad \frac{1+\tan\theta}{\sin\theta} = \frac{1+\tan 45^\circ}{\sin 45^\circ}$$

$$= \frac{1+1}{\frac{1}{\sqrt{2}}}$$

ɔr $= 2\sqrt{2}$

Top 10

Most Important Questions

Q. If 5 tan ß = 4, then

$$\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta}$$

(A) 1/3

(B) 2/5

(C) 3/5

(D) 6

SOLUTION

Sol. Option (A) is correct.

Explanation: $5\tan\beta = 4$

$$\Rightarrow \quad \tan\beta = \frac{4}{5}$$

$$\therefore \quad \frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} = \frac{5\tan\beta - 2}{5\tan\beta + 2}$$

[dividing cos β by Nr. and Dr.]

$$= \frac{5\times\frac{4}{5} - 2}{5\times\frac{4}{5} + 2} = \frac{2}{6} = \frac{1}{3}.$$

Top 10

Most Important Questions

Q. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

(A) 1

(B) 3

(C) 2

(D) 4

SOLUTION

Sol. Option (B) is correct.

Explanation:

$$\because \quad \sin\theta + \cos\theta = \sqrt{2}$$

Squaring on both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 2$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2}$$

But $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= 2$$

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): cot A is the product of cot and A. Reason (R): The value of sin increases as increases.

SOLUTION

. Option (D) is correct.

Explanation: In case of assertion:

cot A is not the product of cot and A. It is the cotangent of $\angle A$.

$\therefore$ Assertion is false.

In case of reason:

The value of $\sin\theta$ increases as θ increases in interval of $\theta° < \theta° < 90°$

$\therefore$ Reason is true:

Hence, assertion is false but reason is true.

Top 10

Most Important Questions

Q.Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If $x = 2 \sin^2\theta$ and $y = 2 \cos^2\theta+1$ then the value of $x + y = 3$.

Reason (R): If $\tan a = 5/12$ then the value of $\sec a$ is $13/12$

SOLUTION

Option (B) is correct.

Explanation: In case of assertion:

We have $x = 2\sin^2\theta$

and $y = 2\cos^2\theta + 1$

Then,
$$x + y = 2\sin^2\theta + 2\cos^2\theta + 1$$
$$= 2(\sin^2\theta + \cos^2\theta) + 1$$
$$= 2 \times 1 + 1 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= 2 + 1 = 3.$$

$\therefore$ Assertion is true.

In case of reason:

$$\tan\alpha = \frac{5}{12}$$

Unsing identity; $\sec^2\alpha - \tan^2\alpha = 1$

$$\sec^2\alpha = 1 + \tan^2\alpha$$

$$\Rightarrow \qquad \sec^2\alpha = 1 + \left(\frac{5}{12}\right)^2$$
$$= 1 + \frac{25}{144}$$
$$= \frac{144 + 25}{144}$$
$$= \frac{169}{144}$$
$$\sec\alpha = \sqrt{\frac{13^2}{12^2}}$$
$$= \frac{13}{12}$$

$\therefore$ Reason is also true.

Hence, both assertion and reason are true but reason is not the correct explanation for assertion.

Q. Assertion : The value of sin $\theta = 4/3$ is not possible.
Reason : Hypotenuse is the largest side in any right angled triangle

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

$$\sin\theta = \frac{P}{H} = \frac{4}{3}$$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.
Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
Thus (a) is correct option.

Q. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is

[SQP 2022]

a) 0°

b) 90°

c) 60°

d) 30°

Solution

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ$ ...(1)

and $\cos\beta = 0$

$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ$ ...(2)

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

Q. Assertion : The value of sin $\theta = 4/3$ is not possible.
Reason : Hypotenuse is the largest side in any right angled triangle

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

Both assertion and reason are true and reason is the correct explanation of assertion.

Let us, consider sinθ = $\frac{4}{3}$

We know that,
sinθ = $\frac{\text{Height}}{\text{Hypotnuse}}$

Now, considering the above data,
Height = 4 unit and Hypotnuse = 3 unit

We also kow that, the triangle must be right angled triangle to get the value of sinθ

So, now apply the pythagoras theorem to get the base of the triangle,
we get base = $\sqrt{3^2 - 4^2}$
$\sqrt{9 - 16}$
$\sqrt{-7}$
Which is not possible.

Hence, assertion is true and to follow the pythagoras theorem the hypotnuse has to be the largest side in the triangle.

Therefore, reason is also correct and reason is the correct explanation for the assertion.

Q. Assertion : $\sin^2 67° + \cos^2 67° = 1$.

Reason : For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We have

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2 67^\circ + \cos^2 67^\circ = 1$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Q. Assertion : The value of sin 60° cos 30° + sin 30° cos 60° is 1.

Reason(R): sin 90° = 1 and cos 90° = 0.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We know that sin 90° = 1 and cos 90° = 0.
So, Reason is correct
For Assertion, we have, sin 60° cos 30° + sin 30° cos 60°
= (√3/2) (√3/2) + (1/2) (1/2)
= 3/4 + 1/4 = 4/4 = 1
So, Assertion is also correct.
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

Q. Assertion : The value of $2\tan^2 45° + \cos^2 30° - \sin^2 60°$ is $\sqrt{2}$.

Reason(R): Value of $\tan 45° = 1$, $\cos 30° = \sqrt{3}/2$ and $\sin 60° = \sqrt{3}/2$.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We know that $\tan 45° = 1$,
$\cos 30° = \sqrt{3}/2$ and $\sin 60° = \sqrt{3}/2$.
So, Reason is correct
For Assertion, we have,
$2 \tan^2 45° + \cos^2 30° - \sin^2 60°$
$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$
$= 2 + 3/4 - 3/4 = 2.$
So, Assertion is False.
Correction option is (d).

Q. Assertion : The value of $\dfrac{2\tan 30^o}{1+\tan^2 30^o}$ is $\dfrac{\sqrt{3}}{2}$

Reason(R): cot A is not defined for A = 0°.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

For Reason, we know that cot A is not defined for A = 0°.
So, Reason is correct
For Assertion, we have

$$\frac{2\tan 30^o}{1+\tan^2 30^o} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

So, Assertion is also correct
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

Top 10

Most Important Questions

Q.The angle of depression of a car parked on the road from the top of 150 m high tower is 30°. The distance of the car from the tower (in metres) is:

(A) $50\sqrt{3}$ m

(B) $150\sqrt{2}$ m

(C) $150\sqrt{3}$ m

(D) 75 m

SOLUTION

Given: Height = 150 m

Angle of depression = 30°

Consider the diagram,

In ΔABC,

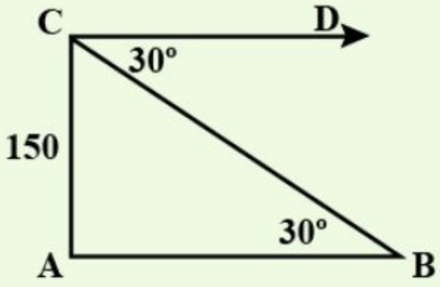
Let, distance of the car from tower BC = x m,

In ΔABC, $\tan 30^\circ = \dfrac{AC}{AB}$

So, $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$

Therefore, AC = $150\sqrt{3}$

So, distance between the tower and car is $150\sqrt{3}$ m



Top 10

Most Important Questions

Q. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the Sun at that time is:

(A) $30°$

(B) $45°$

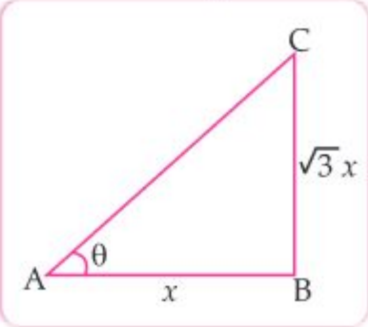
(C) $60°$

(D) $75°$

SOLUTION

Sol. Option (B) is correct.

Explanation: Let the length of shadow is x,



Then height of pole $= \sqrt{3}\,x$

$$\tan\theta = \frac{CB}{AB}$$

$$\tan\theta = \frac{\sqrt{3}x}{x}$$

$$\tan\theta = \sqrt{3}$$

$$\tan\theta = \tan 60^\circ$$

$$\theta = 60^\circ$$

Top 10

Most Important Questions

Q. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. The angle made by the ladder with the horizontal is:

(A) 60°

(B) 45°

(C) 30°

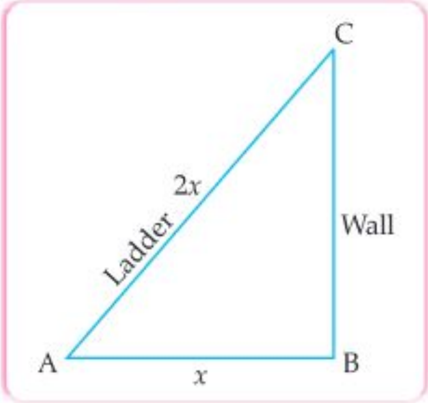
(D) 22.5° [CBSE SA-II, 2015]

SOLUTION

Sol. Option (A) is correct.

Explanation: Let the distance between the foot of the ladder and the wall, AB be x.

Then AC, the length of the ladder $= 2x$



In ΔABC, $\angle B = 90°$

$$\cos A = \frac{x}{2x}$$

$$\Rightarrow \quad \cos A = \frac{1}{2} = \cos 60° \Rightarrow \angle A = 60°$$

Top 10

Most Important Questions

Q. Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is 75 m.

Reason (R): If the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°, then the height of the tower is $10\sqrt{3}$ m.

SOLUTION

Ans. Option (D) is correct.

Explanation: In case of assertion:

Let AW be the ladder and WL = x be the height of wall.

In ΔAWL,

$$\cos 60° = \frac{x}{15}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{x}{15}$$

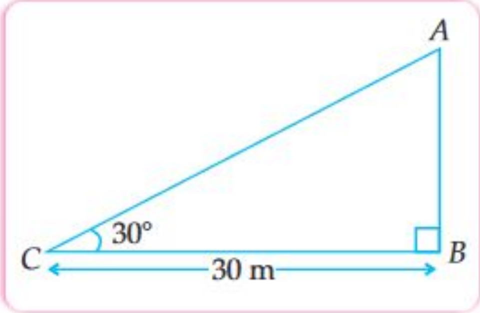
$$\Rightarrow \quad x = 7.5$$

Hence, the required height of the wall is 7.5 m.

∴ Assertion is false.

In case of reason:

Let AB be the tower.



In ΔABC,

$$\tan 30° = \frac{AB}{30}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\Rightarrow \quad AB = 10\sqrt{3}$$

Hence, the required height of the tower is $10\sqrt{3}$ m.

∴ Reason is true.

Hence, assertion is false but reason is true.

Top 10

Most Important Questions

Q.Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If the ratio of the length of a vertical rod and the length of its shadow is 1: $\sqrt{3}$, then the angle of elevation of the sun at that moment is 30°.

Reason (R): If the ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}$:1, then the angle of elevation of the sun is 60°.

SOLUTION

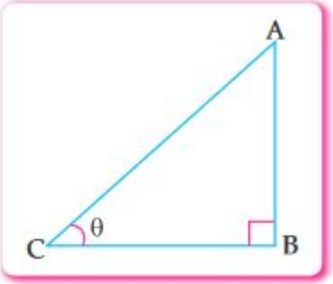
Ans. Option (B) is correct.

Explanation: In case of assertion:

Let AB be a vertical rod and BC be its shadow.

From the figure, $\angle ACB = \theta$.

In ΔABC,



$$\tan\theta = \frac{AB}{BC}$$

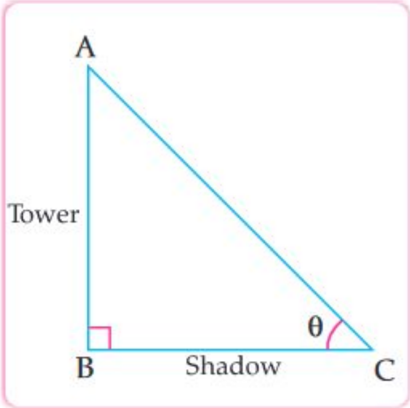
$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}} \text{(Given)}\right]$$

$$\Rightarrow \quad \tan\theta = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

$\therefore$ Assertion is true.

In case of reason:



Let the height of tower be AB and its shadow be BC.

$$\therefore \quad \frac{AB}{BC} = \tan\theta$$

$$= \frac{\sqrt{3}}{1}$$

$$= \tan 60^\circ$$

Hence, the angle of elevation of Sun = 60°.

$\therefore$ Reason is true.

Hence, both assertion and reason are true but reason is not the correct explanation for assertion.

Top 10

Most Important Questions

Q.At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is:

(A) 4 cm

(B) 6 cm

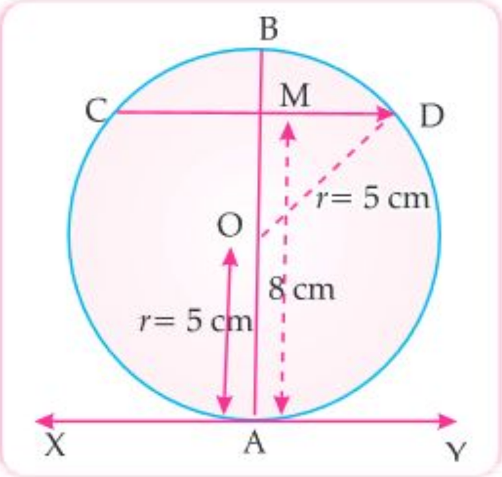
(C) 5 cm

(D) 8 cm

SOLUTION

Sol. Option (D) is correct.

Explanation: XAY is tangent and AO is radius at point of contact A of circle.



$$AO = 5 \text{ cm}$$

$$\therefore \quad \angle OAY = 90^\circ$$

CD is another chord at distance (perpendicular) of 8 cm from A and $CMD \parallel XAY$ meets AB at M.

Join OD.

$$OD = 5 \text{ cm}$$

$$OM = 8 - 5 = 3 \text{ cm}$$

$$\angle OMD = \angle OAY = 90^\circ$$

(co-interior angles)

Now, in right angled ΔOMD

$$MD^2 = OD^2 - MO^2$$

$$= 5^2 - 3^2$$

$$= 25 - 9$$

$$= 16$$

$$\Rightarrow \quad MD = 4 \text{ cm}$$

We know that, perpendiculars from centre O of circle bisect the chord.

$$\therefore \quad CD = 2MD$$

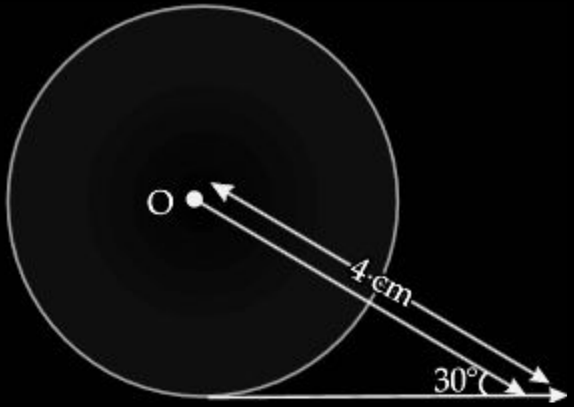
$$= 2 \times 4$$

$$= 8 \text{ cm.}$$

Hence, length of chord, $CD = 8$ cm.

Top 10

Most Important Questions



Q. In the given figure, AT is a tangent to the circle with centre 'O' such that OT = 4 cm and OTA = 30°. Then AT is equal to:

(A) 4 cm

(B) 2√3 cm

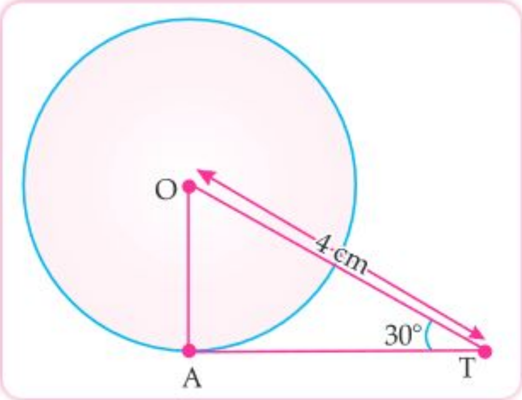
(C) 2 cm

(D) 4√3 cm

SOLUTION

Sol. Option (C) is correct.

Explanation: Join OA. OA is radius and AT is tangent at contact point A.



$\therefore \quad \angle OAT = 90°,$

Given that, $\quad OT = 4 \text{ cm}$

Now, $\quad \frac{AT}{4} = \frac{\text{base}}{\text{hypotenuse}} = \cos 30°$

$\Rightarrow \quad AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3} \text{ cm.}$

Top 10

Most Important Questions

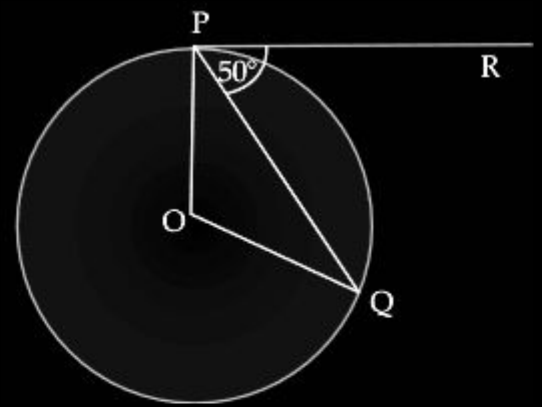
Q. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal to:

(A) 100°

(B) 80°

(C) 90°

(D) 75°



SOLUTION

Sol. Option (A) is correct.

Explanation: OP is radius and PR is tangent at P.

So, $\angle OPR = 90°$

$\Rightarrow \angle OPQ + 50° = 90°$

$\Rightarrow \angle OPQ = 90° - 50°$

$\Rightarrow \angle OPQ = 40°$

In ΔOPQ, $OP = OQ$ (Radii of same circle)

$\therefore \angle Q = \angle OPQ = 40°$

(Angles opposite to equal sides are equal)

Now, $\angle POQ = 180° - \angle P - \angle Q$

$= 180° - 40° - 40°$

$= 180° - 80° = 100°$

$\Rightarrow \angle POQ = 100°.$

Top 10

Most Important Questions

Q. Mark the correct choice as:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is 120°.

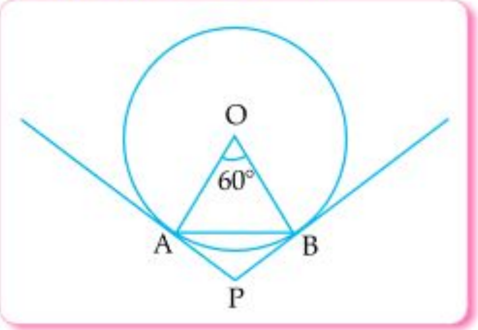
Reason (R): In ΔAOB, OAB is 60°.

SOLUTION

Ans. Option (A) is correct.

Explanation: In case of assertion:

Chord AB subtends $\angle 60°$ at O.



$\therefore \quad \angle OAP = 90°$

Similarly, $\quad \angle OBP = 90°$

In quadrilateral *OAPB*,

$\angle O + \angle P + \angle OAP + \angle OBP = 360°$

$\Rightarrow \quad 60° + \angle P + 90° + 90° = 360°$

$\Rightarrow \quad \angle P = 360° - 240°$

$\Rightarrow \quad \angle P = 120°$

$\therefore$ Assertion is true.

In case of reason:

In ΔAOB,

$\angle AOB = 60°$ (Given)

$\because \quad OA = OB$ (Radii)

$\therefore \quad \angle OAB = \angle OBA$ (Isosceles property)

$\angle AOB + \angle OAB + \angle OBA = 180°$

(Angle sum property)

$60° + 2\angle OAB = 180°$

$2\angle OAB = 120°$

$\angle OAB = 60°$.

So, reason is also true.

Top 10

Most Important Questions

Q.Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

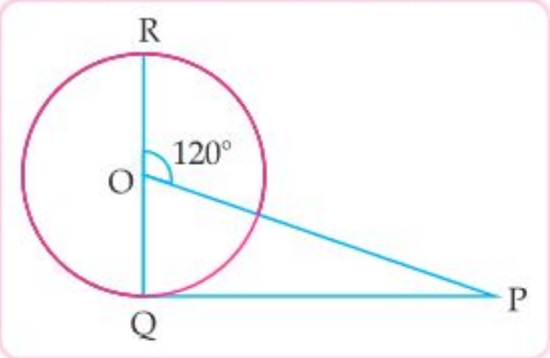
Assertion (A): PQ is a tangent drawn from an external point P to a circle with centre O. QOR is the diameter of the circle. If $\angle POR = 120°$, then the measure of $\angle OPQ$ is $30°$.

Reason (B): In Triangle POQ, $\angle POQ = 60°$

SOLUTION

Ans. Option (A) is correct.

Explanation: In case of assertion:



Given that,

PQ is a tangent, $\angle POR = 120°$

$$\angle POR = \angle OQP + \angle OPQ$$

(Exterior angle sum property)

$$\angle OPQ = 120° - 90° = 30°$$

$$\angle OPQ = 30°$$

Assertion is true.

In case of reason:

Since ROQ is a straight line.

$$\angle ROP = 120° \quad \text{(given)}$$

$$\therefore \quad \angle POQ = 180° - 120°$$

$$= 60°$$

∴ Reason is true.

Q. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240^0. If another circle of the area same as the sector is formed, then radius of the new circle is

[Board 2022 Term 1 SQP STD]

a) 79.5 cm

b) 81.6 cm

c) 83.4 cm

d) 88.5 cm

Solution

$$\text{Area of sector} = \frac{240^\circ}{360^\circ} \times \pi(100)^2 = 20933 \text{ cm}^2$$

Let r be the radius of the new circle, then

$$20933 = \pi r^2$$

$$r = \sqrt{\frac{20933}{\pi}} = 81.6 \text{ cm}$$

Thus (b) is correct option.

Q. If the circumference of a circle increases from 4 π to 8 π , then its area is

[Board 2022 Term 1 STD]

a Halved

b Doubled

c Tripled

d Quadrupled

Solution

$$2\pi r = 4\pi \Rightarrow r = 2$$

$$\text{Area} = \pi(2)^2 = 4\pi$$

When, $$2\pi r = 8\pi \Rightarrow r = 4$$

$$\text{Area} = 16\pi$$

Thus area is quadrupled.
Thus (d) is correct option.

Q. If the perimeter of a semi-circular protractor is 36 cm, then its diameter is

[Board 2022 Term 1 STD]

a) 10 cm

b) 14 cm

c) 12 cm

d) 16 cm

Solution

$$\text{Perimeter} = \frac{2\pi r}{2} + 2r = \pi r + 2r$$

$$(\pi + 2)r = 36$$

$$\left(\frac{36}{7}\right) - r = 36 \Rightarrow r = 7\text{ cm}$$

Hence, diameter $2r = 7 \times 2 = 14\text{ cm}$

Thus (b) is correct option.

Q. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

a) 22:7

b) 14:11

c) 7:22

d) 11:14

Solution

Let radius of circle be r and side of a square be a.
According to the given condition,

$$\text{Perimeter of a circle} = \text{Perimeter of a square}$$

$$2\pi r = 4a$$

$$a = \frac{\pi r}{2} \quad \text{...(1)}$$

Now, $$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{(\pi \frac{r}{2})^2} \quad \text{[from Eq. (1)]}$$

$$= \frac{\pi r^2}{\pi^2 \frac{r^2}{4}} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}}$$

$$= \frac{28}{22} = \frac{14}{11}$$

Hence, the required ratio is 14:11.
Thus (b) is correct option.

Q. The area of the square that can be inscribed in a circle of radius 8 cm is

a) $256\ cm^2$

b) $128\ cm^2$

c) $64\sqrt{2}\ cm^2$

d) $64\ cm^2$

Solution

Radius of circle, $r = 8 \text{ cm}$

Diameter of circle, $d = 2r = 2 \times 8 = 16 \text{ cm}$

Since, square inscribed in circle.

$$\text{Diagonal of square} = \text{Diameter of circle}$$

Now,
$$\text{Area of square} = \frac{(\text{Diagonal})^2}{2} = \frac{(16)^2}{2} = \frac{256}{2}$$
$$= 128 \text{ cm}^2$$

Thus (b) is correct option.

Q. The area of a circular path of uniform width d surrounding a circular region of radius r is

a. $\pi d(d + 2r)$

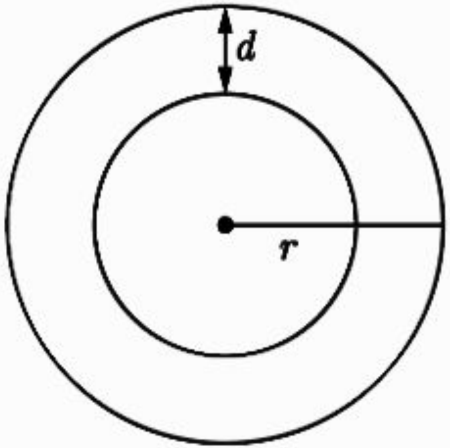
b. $\pi(2r + d)r$

c. $\pi(r + d)r$

d. $\pi(2r + d)d$

Solution

$$\text{Required area} = \pi\left[(r+d)^2 - r^2\right]$$
$$= \pi[r^2 + d^2 + 2rd - r^2]$$
$$= \pi[d^2 + 2rd] = \pi d[d + 2r]$$



Thus (a) is correct option.

Q. In a circle of radius 14 cm, an arc subtends an angle of 45^0 at the centre, then the area of the sector is

a. 71 cm^2

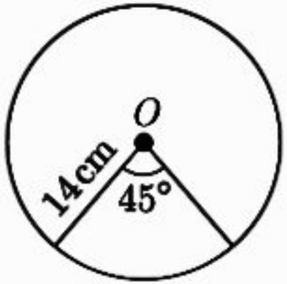
b. 76 cm^2

c. 77 cm^2

d. 154 cm^2

Solution

Given, $r = 14\text{ cm}$ and $\theta = 45^\circ$



$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{8} \times 22 \times 2 \times 14 = 77\text{ cm}^2$$

Thus (c) is correct option.

Q. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

a) 31 cm

b) 25 cm

c) 62 cm

d) 50 cm

Solution

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$$R^2 = r_1^2 + r_2^2$$

$$= 24^2 + 7^2 = 625$$

$$R = \sqrt{625} = 25 \text{ cm}$$

Diameter of a circle

$$2R = 2 \times 25 = 50 \text{ cm}$$

Thus (d) is correct option.

Q. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by

[Board 2020 OD Standard]

a) 200%

b) 500%

c) 700%

d) 800%

Solution

Let r be the original radius of sphere. If we increased radius by 100 %. it will be $2r$.

$$V_r = \frac{4}{3}\pi r^3$$

Now $$V_{2r} = \frac{4}{3}\pi \times (2r)^3 = \frac{4}{3}\pi \times 8r^2$$

Thus new volume is 8 times of original volume.

Hence when the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

Thus (c) is correct option.

Q. If the perimeter of one face of a cube is 20 cm, then its surface area is

[Board 2019 OD]

a) $120\ cm^2$

b) $150\ cm^2$

c) $125\ cm^2$

d) $400\ cm^2$

Solution

Edge of cube, $a = \frac{20}{4}$ cm $= 5$ cm

Surface area $6a^2 = 6 \times 5^2$ cm^2 $= 150$ cm^2

Thus (b) is correct option.

Q. Volumes of two spheres are in the ratio 64 : 27. The the ratio of their surface areas is

a) 3 : 4

b) 4 : 3

c) 9 : 16

d) 16 : 9

Solution

Let the radii of the two spheres are r_1 and r_2, respectively.
Given, ratio of their volumes,

$$V_1 : V_2 = 64 : 27$$

$$\frac{V_1}{V_2} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{64}{27}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Now, ratio of their surface area,

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the required ratio of their surface area is 16 :9.
Thus (d) is correct option.

Q. Ratio of lateral surface areas of two cylinders with equal height is

a) $1 : 2$

b) $H : h$

c) $R : r$

d) None of these

Solution

$$2\pi Rh : 2\pi rh = R : r$$

Thus (c) is correct option.

Q. A 20 m deep well, with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. The height of the platform is

a. 2.5 m

b. 3.5 m

c. 3

d. 2 m

Solution

$$\text{Radius of the well} = \frac{7}{2}\text{m} = 3.5\,\text{m}$$

$$\text{Volume of the earth dug out} = \frac{22}{7} \times (3.5)^2 \times 20$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 20$$

$$= 770\,\text{m}^3$$

$$\text{Area of platform} = (22 \times 14)\,\text{m}^2$$

$$= 308\,\text{m}^2$$

$$\text{Height} = \frac{770}{308} = 2.5\,\text{m}$$

Q. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then the volume of remaining solid is

a) $280\pi\ cm^3$

b) $330\pi\ cm^3$

c) $240\pi\ cm^3$

d) $440\pi\ cm^3$

Solution

Volume of the remaining solid

$$= \text{Volume of the cylinder} - \text{Volume of the cone}$$

$$= \pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10$$

$$= (360\pi - 120\pi) = 240\pi \text{ cm}^3$$

Thus (c) is correct option.

Q. The diameter of hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?

a. 50 %

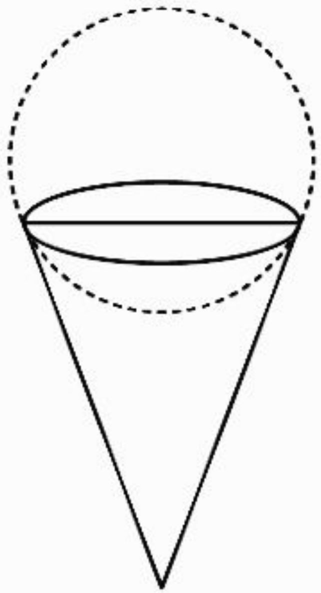
b. Less than 50 %

c. More than 50 %

d. 100 %

Solution

Though it is given that diameter of the cone is equal to the diameter of the spherical ball. But the ball will not fit into the cone because of its slant shape. Hence more than 50% of the portion of the ball will be outside the cone.



Thus (c) is correct option.

Q. A right circular cylinder of radius r and height h (where, $h > 2r$) just encloses a sphere of diameter

a. r

b. $2r$

c. h

d. $2h$

Solution

Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is $2r$.

Thus (b) is correct option.

Q. Ratio of volumes of two cones with same radii is

a) $h_1 : h_2$

b) $s_1 : s_2$

c) $r_1 : r_2$

d) None of these

Solution

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2 \qquad (r_1 = r_2)$$

$$h_1 : h_2$$

Thus (a) is correct option.

Q. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

a) $4\pi r^2$

b) $6\pi r^2$

c) $3\pi r^2$

d) $8\pi r^2$

Solution

Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$

Thus (a) is correct option.

Q. Assertion : If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is 40 cm^2.
Reason : Circumference of the circle = length of the wire

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

We have $2\pi r = 22$

$r = 3.5\,\text{cm}$

$$\text{Area of the circle} = \frac{22}{7} \times 3.5 \times 3.5$$
$$= 38.5\,\text{cm}^2$$

Assertion is not correct, but reason is true.

Thus (d) is correct option.

Q. Assertion : In a circle of radius 6 cm, the angle of a sector 60c . Then the area of the sector is $18\frac{6}{7}$ cm^2 .

Reason : Area of the circle with radius r is πr^2.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

$$\text{Area of the sector } = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} = 18\frac{6}{7}\text{cm}^2.$$

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

Q. Assertion : If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason : If *r* be the radius and *h* be the slant height of the cone, then slant height = $\sqrt{h^2 + r^2}$

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

Slant height $l = \sqrt{\left(\frac{14}{2}\right)^2 + (24)^2}$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625} = 25$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Q. Assertion : Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.

Reason : : If r be the radius and h be the height of the cylinder, then total surface area = $2\pi rh + 2\pi r^2$.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

Total surface area,

$$2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 14(30 + 14) = 88(44)$$

$$= 3872 \text{ cm}^2$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Q. Assertion : If the outer and inner diameter of a circular path is 10 m and 6 m then area of the path is 16π m^2

Reason : If R and r be the radius of outer and inner circular path, then area of path is $\pi(R^2 - r^2)$.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Solution

$$\text{Area of the path} = \pi\left[\left(\frac{10}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right]$$

$$= \pi(25 - 9) = 16\,\pi$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Top 10

Most Important Questions

Q. The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is:

(A) 48 cm^2

(B) 72 cm^2

(C) 96 cm^2

(D) 108 cm^2

$$tsa = sum^2 - d^2$$

SOLUTION

II Method (Trick)

$$tsa = sum^2 - d^2$$

$$tsa = (6\sqrt{3})^2 - (2\sqrt{3})^2 = 108 - 12 = 96$$

Ans. Option (C) is correct.

Explanation:

Given: $l + b + h = 6\sqrt{3}$ cm ...(i)

and the length of its diagonal $= 2\sqrt{3}$ cm

i.e., $\sqrt{l^2 + b^2 + h^2} = 2\sqrt{3}$

Squaring both sides, we get

$$l^2 + b^2 + h^2 = 12 \quad \text{...(ii)}$$

From eq. (i),

$$(l + b + h)^2 = (6\sqrt{3})^2$$

$$\Rightarrow l^2 + b^2 + h^2 + 2(lb + bh + hl) = 108$$

$$\Rightarrow 12 + 2(lb + bh + hl) = 108$$

[From eq. (iii)]

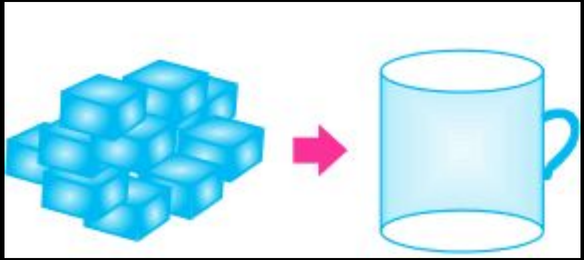
$$\Rightarrow 2(lb + bh + hl) = 96$$

Hence, total surface area of the cuboid is 96 cm^2.

Top 10

Most Important Questions

Q.Anushka melted 11 chocolate cubes in a cylindrical cup as shown.



If the length of the side of each cube is k cm and the radius of the cup is r cm, which of these represents the height of the melted chocolate in the cup?

(A) $7k^3/4r$ (B) $7k^3/2r^2$

(C) $7k^2/4r$ (D) $7k^2/4r^2$

SOLUTION

Sol. Option (B) is correct.

Explanation:

$$\text{Volume of 11 cubes} = \text{Volume of cylinder}$$

$$11 \times (\text{side})^3 = \pi r^2 h$$

$$11 \times k^3 = \frac{22}{7} \times r^2 \times h$$

$$\frac{11k^3 \times 7}{22r^2} = h$$

$$\therefore \quad h = \frac{7k^3}{2r^2}$$

Top 10

Most Important Questions

Q. If the radius of the base of a right circular cylinder is halved, keeping the height same, then the ratio of the volume of the reduced cylinder to that of the original cylinder is-

(A) 1:2

(B) 1:4

(C) 1:16

(D) 2:1

SOLUTION

Ans. 1 : 4

Explanation : Volume of the original cylinder

$$= \pi r^2 h$$

Volume of the reduced cylinder $= \pi\left(\frac{r}{2}\right)^2 h,$

$$= \frac{\pi}{4} r^2 h.$$

So required ratio

$$= \frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}}$$

$$= \frac{\frac{\pi}{4} r^2 h}{\pi r^2 h} = \frac{1}{4}$$

Q. Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): In a right circular cone, the cross-section made by a plane parallel to the base is a circle.

Reason (R): If the volume and the surface area of a solid hemisphere are numerically equal, then the diameter of hemisphere is 9 units.

SOLUTION

Ans. Option (B) is correct.

Explanation: In case of assertion:

In a right circular cone, if any cut is made parallel to its base, we get a circle.

$\therefore$ Assertion is true.

In case of reason:

Let radius of sphere be r.

Given, volume of hemisphere

$$= \text{Surface area of hemisphere}$$

or, $$\frac{2}{3}\pi r^3 = 3\pi r^2$$

or, $$r = \frac{9}{2} \text{ units}$$

$\therefore$ $$\text{Diameter} = \frac{9}{2} \times 2$$

$$= 9 \text{ units}$$

$\therefore$ Reason is also true.

Top 10

Most Important Questions

Q.Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14m and height 24m to a centre for handicapped persons welfare. The slant height of the conical tent is 25.

Reason (R): According to assertion, the surface area of 10 tents is 5500 m^2.

SOLUTION

Ans. Option (A) is correct.

Explanation: For assertion,

$$\text{Radius of tent } (r) = \frac{\text{Diameter}}{2} = 7 \text{ m}$$

and height $(h) = 24$ m

$$\therefore \text{slant height of the tent} = \sqrt{h^2 + r^2}$$

$$= \sqrt{(24)^2 + (7)^2}$$

$$= \sqrt{576 + 49} = 25 \text{ m.}$$

So, assertion is true.

For reason:

$$\text{Surface area of 10 tents} = \pi r l \times 10$$

$$= \frac{22}{7} \times 7 \times 25 \times 10$$

(Proved above, $l = 25$ m)

$$= 5500 \text{ m}^2.$$

So, reason is also true.

Both A and R are true and R is the correct explanation of A.

Top 10

Most Important Questions

Q. If the difference of mode and median of a data is 24, then the difference of median and mean is:

(A) 8

(B) 24

(C) 12

(D) 36

SOLUTION

Ans. *(b)* *12* **[CBSE Marking Scheme SQP Std. 2022]**

Explanation: Since, Mode = 3 Median – 2 Mean

$\Rightarrow$ Mode = 2 Median + Median – 2 Mean

$\Rightarrow$ Mode – Median = 2 Median – 2 Mean

$\Rightarrow$ Mode – Median = 2 (Median – Mean)

Given, difference of mode and median of a data is 24

$\therefore$ 24 = 2 (Median – Mean)

$\Rightarrow$ Median – Mean = 12

Top 10

Most Important Questions

Q. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set:

(A) is increased by 2.

(B) is decreased by 2.

(C) is two times the original median.

(D) remains the same as that of the original set.

SOLUTION

The correct option is **D** remains the same as that of the original set.

Given that number of observations is 9

Therefore median is $\left(\frac{n+1}{2}\right)^{th}$ value $= \frac{10}{2} = 5^{th}$ value

Given that the largest 4 observations is increased by 2

The last four observations are $9^{th}, 8^{th}, 7^{th}, 6^{th}$

Therefore, the median doesn't change.

Top 10

Most Important Questions

Q.While computing mean of grouped data, we assume that the frequencies are:

(A) evenly distributed over all the classes

(B) centred at the class marks of the classes

(C) centred at the upper limits of the classes

(D) centred at the lower limits of the classes

SOLUTION

Sol. Option (B) is correct.

Explanation: In grouping the data from ungrouped data, all the observations between lower and upper limits of class marks are taken in one group then mid-value or class mark is taken for further calculation.

Therefore, frequencies or observations must be centred at the class marks of the classes.

Most Important Questions

Class	f
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	7
185-205	4

Q. Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): Consider the following data: The difference of the upper limit of the median class and the lower limit of the modal class is 20.

Reason (R): The median class and modal class of grouped data always be different.

SOLUTION

. Option (C) is correct.

Explanation: In case of assertion:

Class	Frequency	Cumulative frequency
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	7	63
185 – 205	4	67

Hence, $n = 67$ (odd)

So, Median $= \dfrac{67+1}{2} = 34$

34 lies in class 125–145.

So, median class is 125–145 and upper limit is 145.

Now, the maximum frequency is 20 and it lies in class 125–145 (modal class).

Lower limit of modal class = 125.

Hence, the required difference = 145 – 125 = 20.

$\therefore$ Assertion is true.

In case of reason:

The median and modal class may be same. If modal class is median class which is not always possible as the number of frequencies may be maximum in any class.

So, given statement is not true.

$\therefore$ Reason is false.

Most Important Questions

Q.Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): If the median of a series exceeds the mean by 3, then mode exceeds the mean by 10.

Reason (R): If mode = 12.4 and mean = 10.5, then the median is 11.13.

SOLUTION

Ans. Option (D) is correct.

Explanation: In case of assertion:

Given, median = mean + 3

Since, Mode = 3 Median – 2 Mean

= 3 (Mean + 3) – 2 Mean

$\Rightarrow$ Mode = Mean + 9

Hence, mode exceeds mean by 9.

$\therefore$ Assertion is false.

In case of reason:

$$\text{Median} = \frac{1}{3}\text{ Mode} + \frac{2}{3}\text{ Mean}$$

$$= \frac{1}{3}(12.4) + \frac{2}{3}(10.5)$$

$$= \frac{12.4}{3} + \frac{21}{3}$$

$$= \frac{12.4 + 21}{3} = \frac{33.4}{3}$$

$$= \frac{33.4}{3} = 11.13$$

$\therefore$ Reason is true.

Most Important Questions

Q. For an event E, $P(E) + P(\overline{E}) = x$ then the value of x^3-3 is

(A) -2

(B) 1

(C) 2

(D) -1

SOLUTION

Sol. Option (A) is correct.

Explanation: Given

$$P(E) + P(\bar{E}) = x \qquad \text{...(i)}$$

Also, according to the law of probability,

$$P(E) + P(\bar{E}) = 1 \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$x = 1$$

Put value of x in $x^3 - 3$, we get

$$x^3 - 3 = (1)^3 - 3 = 1 - 3 = -2$$

Most Important Questions

Q.Riya and Kajal are friends. The probability that both will have the same birthday in a non-leap is:

(A) 364/365

(B) 31/365

(C) 1/365

(D) 1/133225

SOLUTION

Ans. *(c)* $\frac{1}{365}$

Explanation: Riya may have any one of the 365 days of the year as her birthday.

Similarly, Kajal may have any one of the 365 days as her birthday.

A total number of ways in which Riya and Kajal may have their birthday = 365 x 365

Then Riya and Kajal may have the same birthday on any one of 365 days.

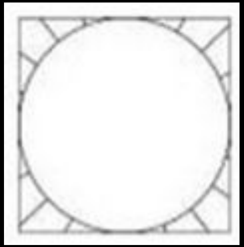
Therefore the probability of Riya and Kajal may have same birthday are:

$$= \frac{365}{365 \times 365} = \frac{1}{365}$$

Top 10

Most Important Questions

Q. There is a square board of side '2a' units circumscribing a red circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on the shaded region is.



(A) $\pi/4$

(B) $(4-\pi)/4$

(C) $(\pi-4)/4$

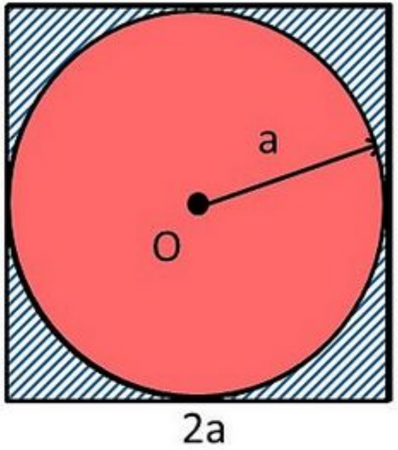
(D) $4/\pi$

SOLUTION

Probability of placing a dot on shaded region

$$= \frac{Area\ of\ shaded\ region\ (blue)}{Total\ area}$$

$$= \frac{\boldsymbol{Area\ of\ square - Area\ of\ circle}}{\boldsymbol{Area\ of\ square}}$$



Area of square

Side = 2a

Area = $(2a)^2$

= $\boldsymbol{4a^2}$ **units**

Area of circle

Radius of circle = $\frac{2a}{2}$ = a

Area of circle = πa^2

SOLUTION

Now,

Probability of placing a dot on shaded region

$$= \frac{Area\ of\ square - Area\ of\ circle}{Area\ of\ square}$$

$$= \frac{4a^2 - \pi a^2}{4a^2}$$

$$= \frac{(4 - \pi)a^2}{4a^2}$$

$$= \frac{4 - \pi}{4}$$

Most Important Questions

Q. Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion (A): The probability of winning a game is 0.345, than the probability of losing it, is 0.655.

Reason (R): P(E) + P (not E) = 1

SOLUTION

Ans. *(a)* *Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation: We have,

$$P(E) = 0.345$$

where E = event of winning

$$P(\text{not } E) = 1 - P(E) = 1 - 0.345 = 0.655$$

Hence, both assertion and reason are true and reason is the correct explanation of assertion.

Most Important Questions

Q.Mark the correct choice as:

(A) Both A and Rare true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Assertion: The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is 14.

Reason: . If the probability of an event is p, the probability of its complementary event will be 1-p.

SOLUTION

Total number of eggs = 400

Probability of getting a bad egg P(E) = 0.035

Consider x as the number of bad eggs

The formula to find the probability is

P(E) = Number of bad eggs/ Total number of eggs

Substituting the values

0.035 = x/400

By further calculation

35/1000 = x/400

x = 35/1000 x 400

x = 140/10

x = 14

Therefore, the number of bad eggs in the lot is 14.

Ans: b) both assertion and reason are correct but reason is not correct explanation for assertion.